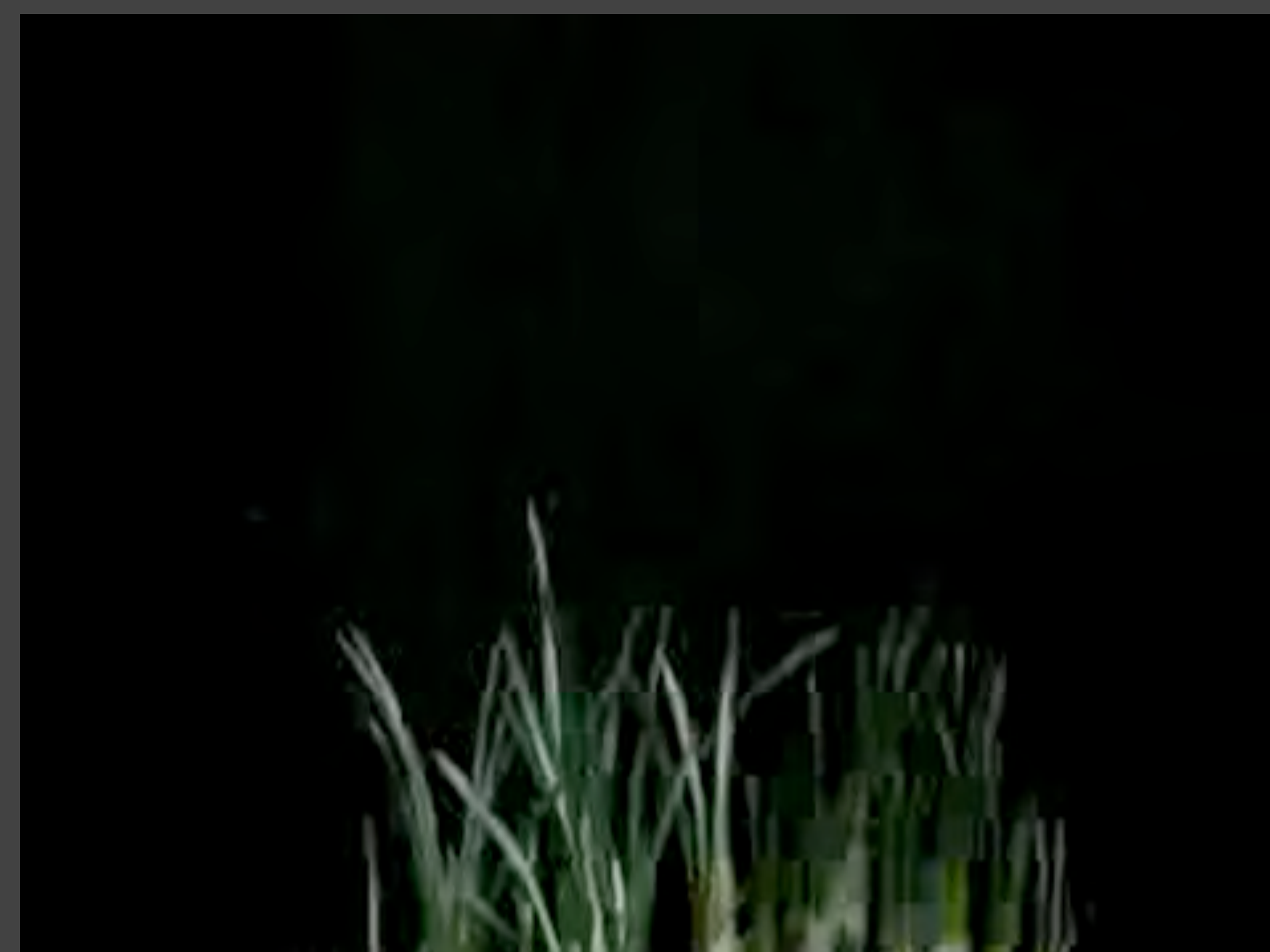
Deciphering Nature's Code

The Secret Mathematics of the Natural World

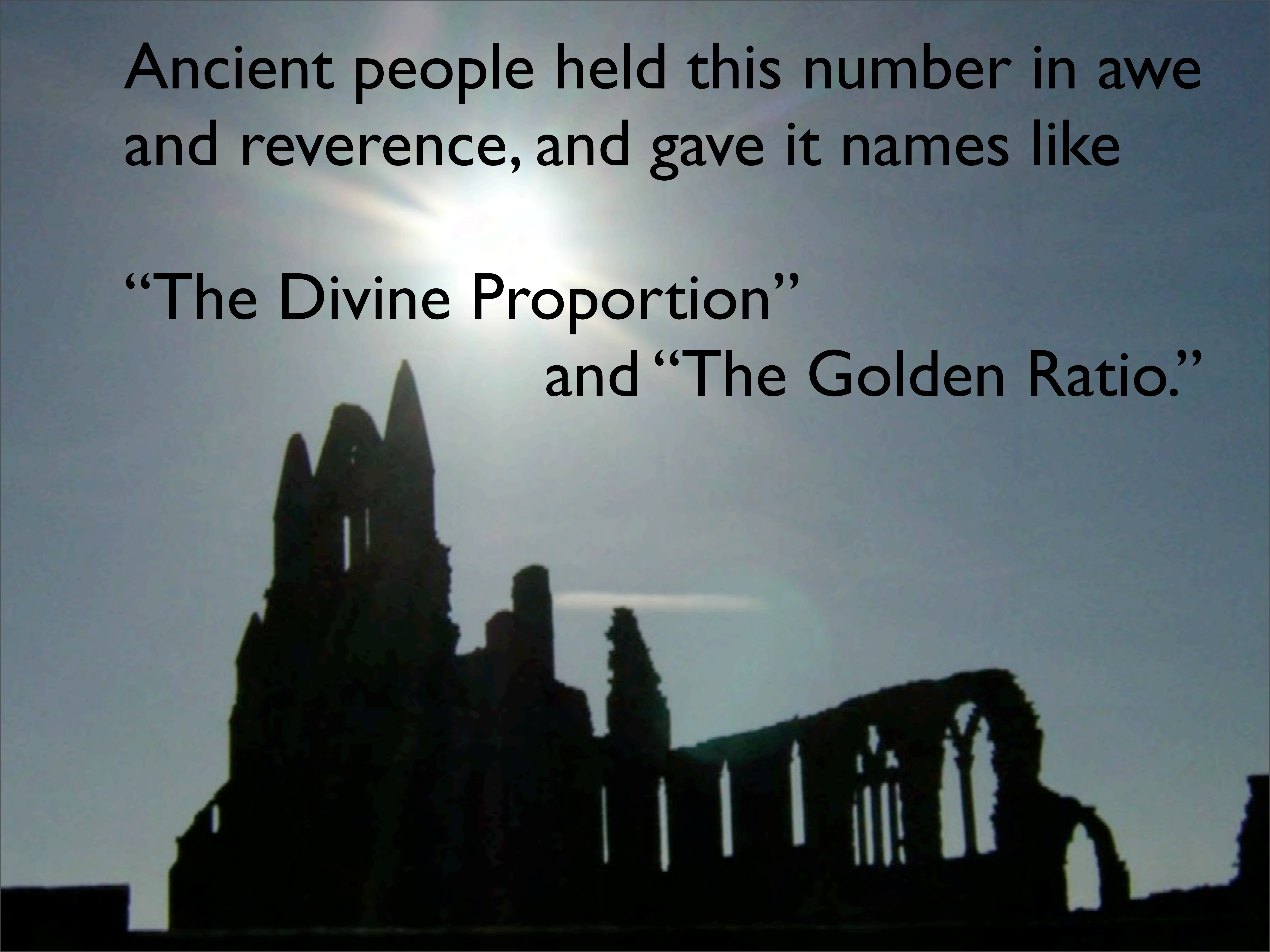
– by Mike Naylor –



Welcome!

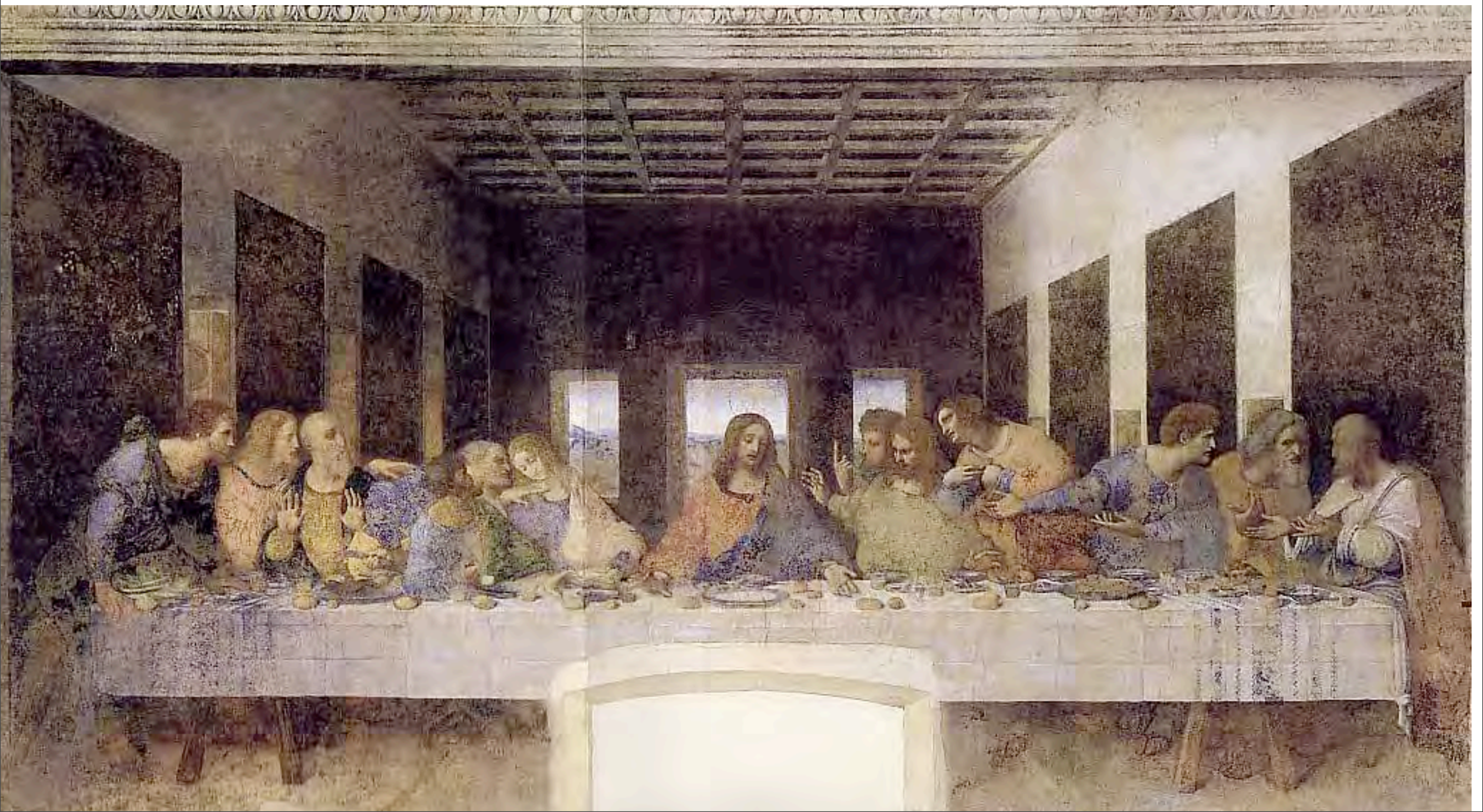

The natural world is alive with
beautiful and amazing shapes.

These forms are all connected by a single number. Mathematics' most mysterious number.


Ancient people held this number in awe and reverence, and gave it names like
“The Divine Proportion”
and “The Golden Ratio.”

It has inspired some of the greatest art and architecture of all time


It is considered to be "the most beautiful number"



It’s found in many places in nature.

What is this number?
Why is it so amazing?

Why does it show up often in nature?

Recent research has answered this question and revealed a deep connection between mathematics and science.

Euclid (c. 300 BCE)

Known as the
"Father of Geometry",
he wrote a math book called
Elements that is still used
today — 2300 years later!

The Divine Proportion

Cut a segment into "mean" and "extreme" ratios.

$$\frac{\text{longer part}}{\text{shorter part}} = \frac{\text{whole thing}}{\text{longer part}}$$

$$\frac{x}{1} = \frac{x+1}{x}$$

$$\frac{x}{1} \cdot x = \frac{x+1}{\cancel{x}} \cdot \cancel{x}$$

$$x^2 = x + 1$$

If you square this number, it's the same as adding 1.

1? $1^2 \stackrel{?}{=} 1 + 1$ 2? $2^2 \stackrel{?}{=} 2 + 1$

3? $3^2 \stackrel{?}{=} 3 + 1$ 1.5 ? $1.5^2 \stackrel{?}{=} 1.5 + 1$

$$2.25 \neq 2.5$$

Hurray for the Quadratic Equation!

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$ $b = -1$ $c = -1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}$$

$$x^2 = x + 1$$

$$\frac{1 \pm \sqrt{5}}{2} = 1.618033988749895....$$

Sooooooooooooooooooooo beautiful!

$$1.618033988749895^2$$

$$= 2.618033988749895$$

$$\frac{x^2}{x} = \frac{x}{x} + \frac{1}{x}$$

$$x = 1 + \frac{1}{x}$$

$$-1 \qquad -1$$

$$x - 1 = \frac{1}{x}$$

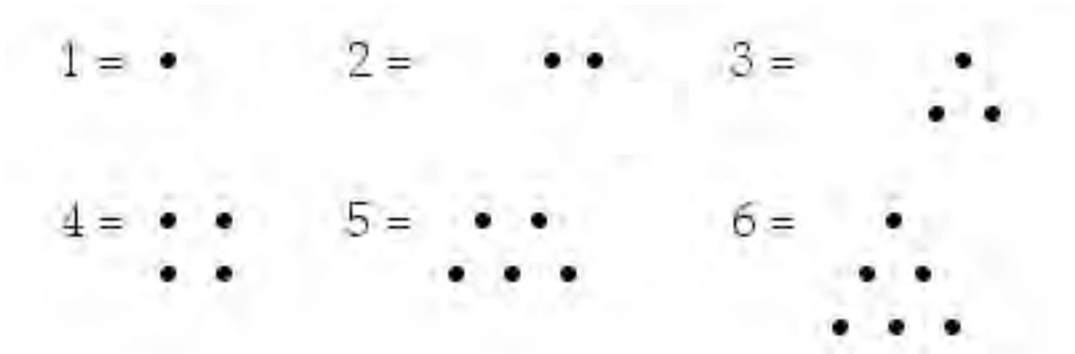
$$\frac{1}{1.618033988749895} = 0.618033988749895$$

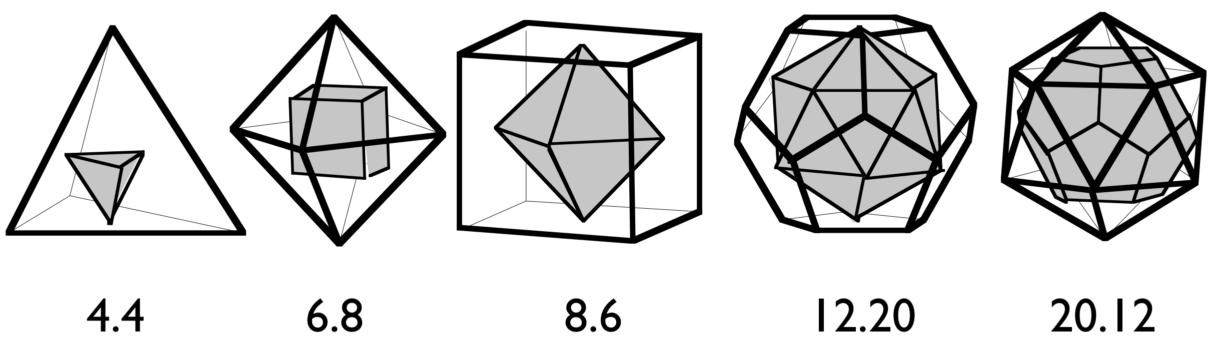
This number is so important, it gets its own letter:

the Greek letter *phi*, written like this: Φ

How can you use a number like 1.618... in art?

And why would you want to?





In ancient times, people thought of numbers as shapes ...

... and Φ makes some of the most amazing shapes.

Φ does number tricks:

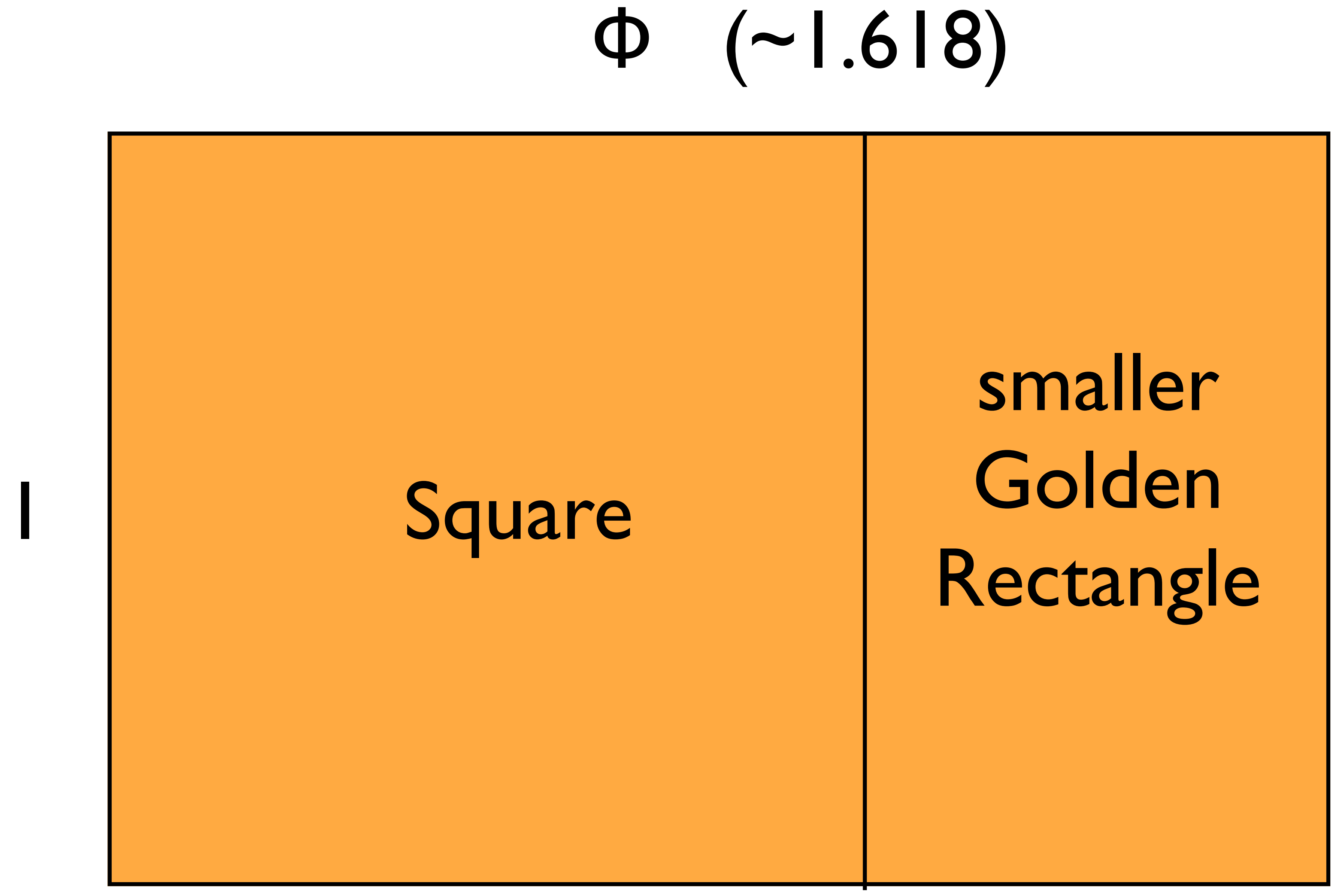
$$\frac{1}{\Phi} = 0.618033988749895 = \Phi - 1$$

$$\Phi = 1.618033988749895$$

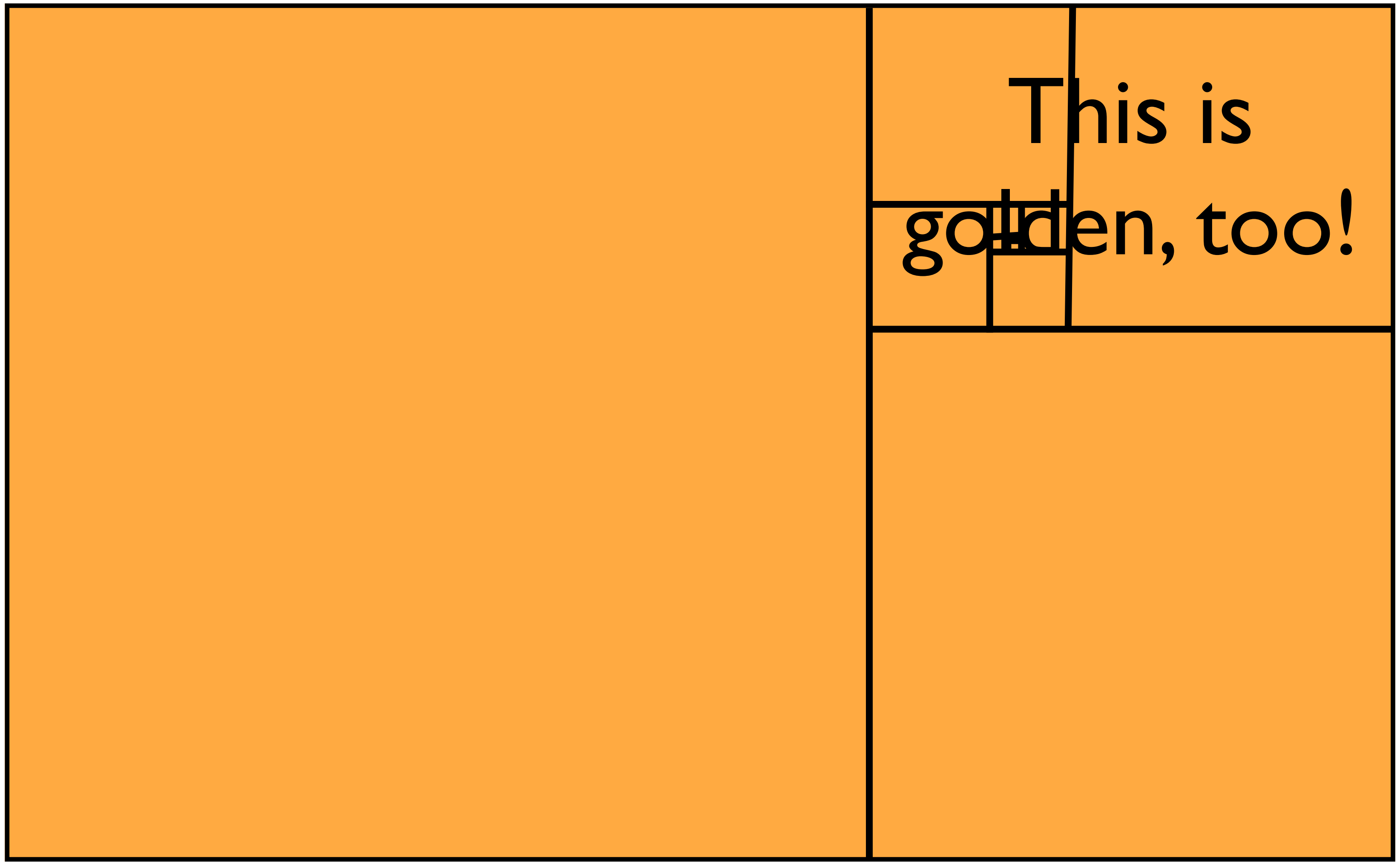
$$\Phi^2 = 2.618033988749895 = \Phi + 1$$

Φ does geometry tricks too....

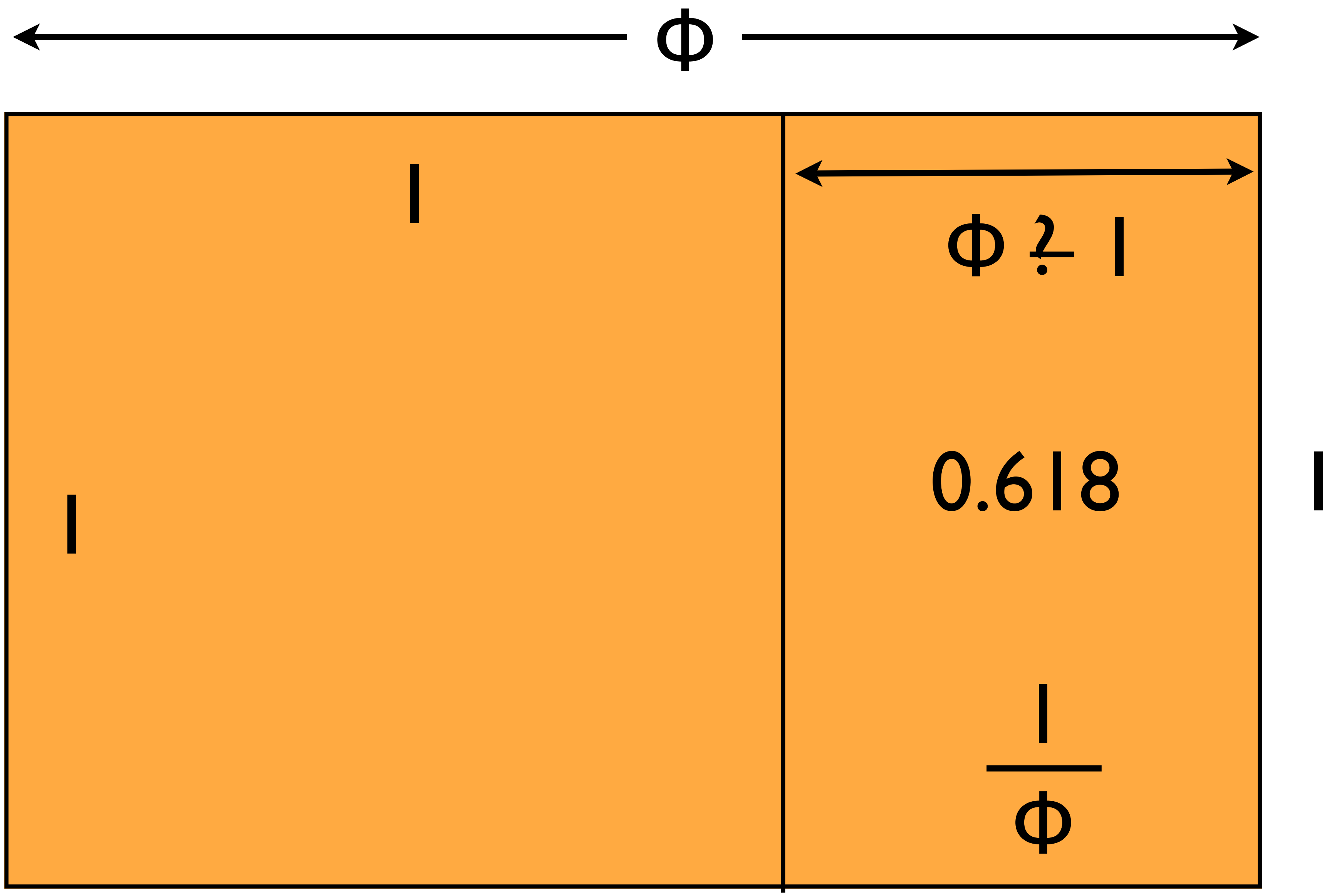
Golden Rectangle



Golden Rectangle



Golden Rectangle



Golden Rectangle

Ratio of $\frac{\text{long}}{\text{short}}$

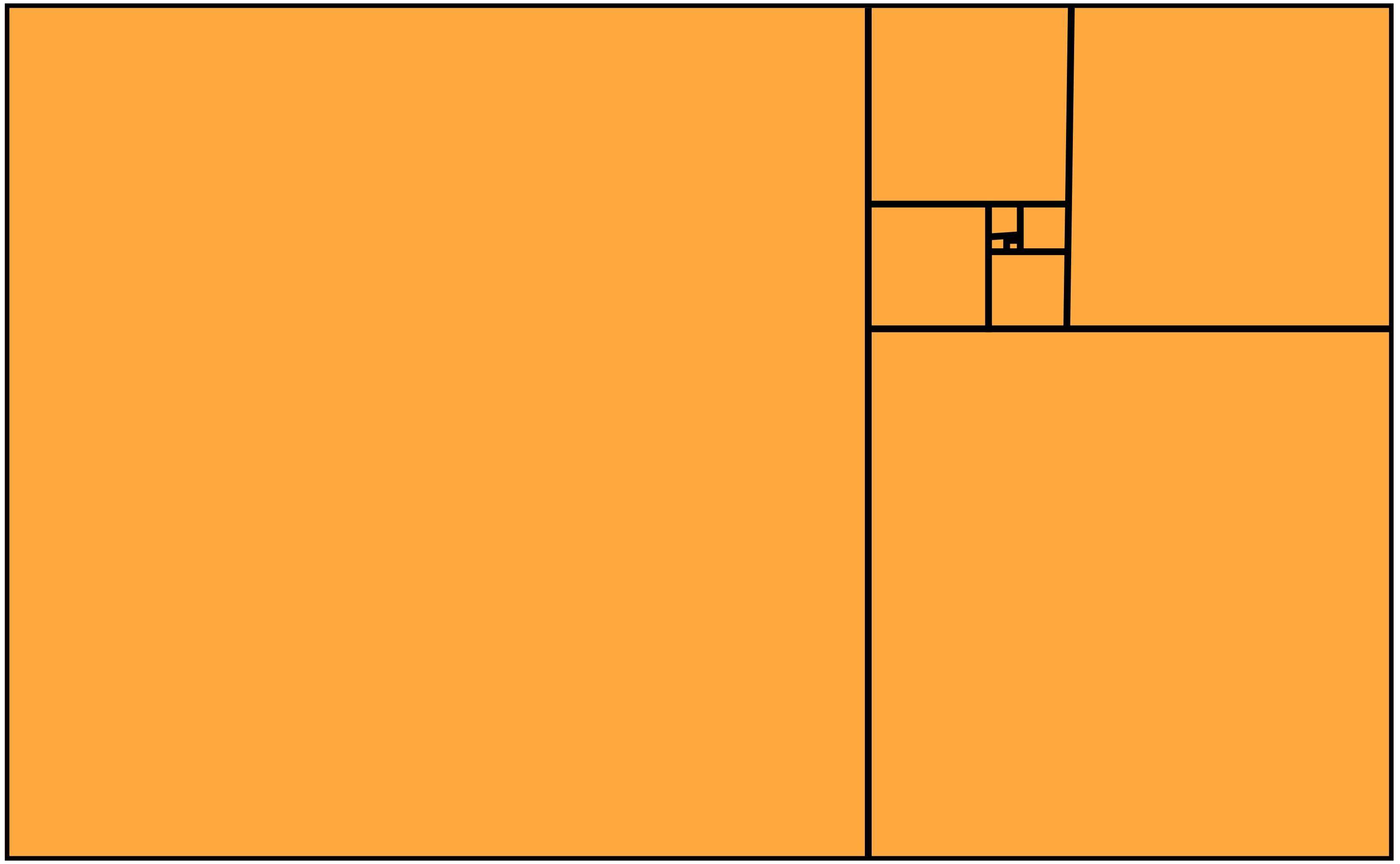
$$= \frac{1}{\frac{1}{\Phi}} = \frac{\Phi}{1}$$

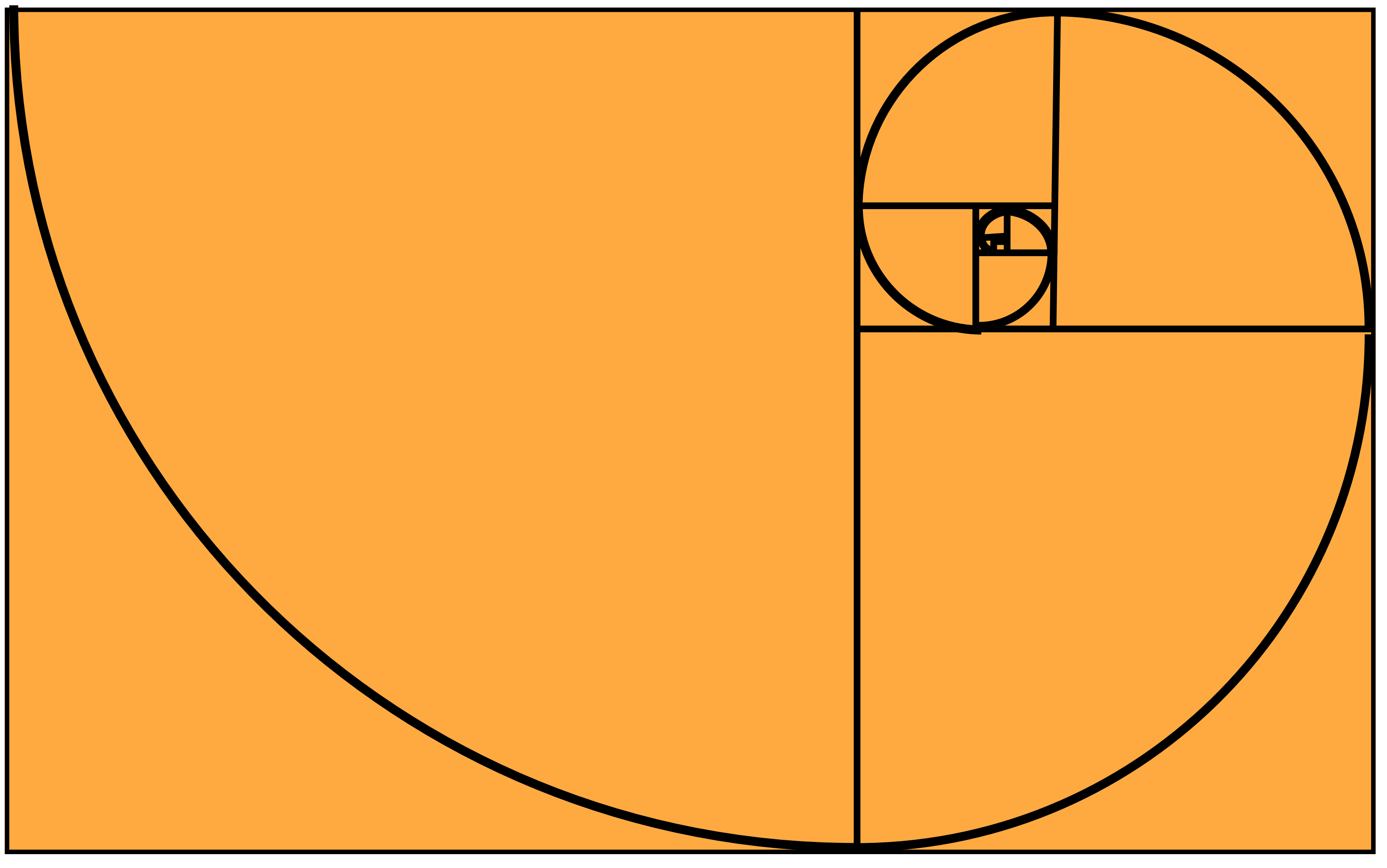
This small rectangle is also golden!

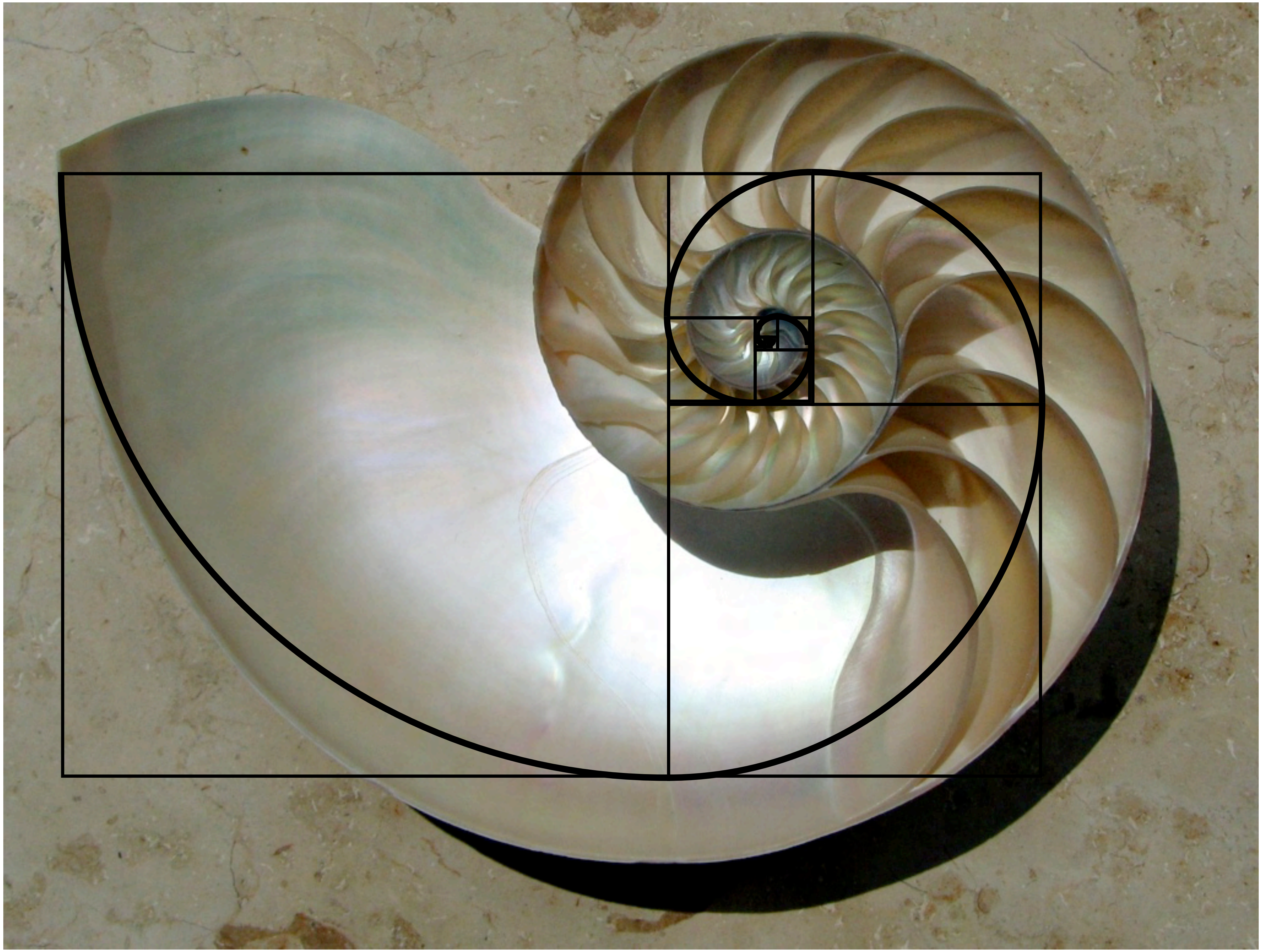
$\frac{1}{\Phi}$

1

short side x Φ = long side

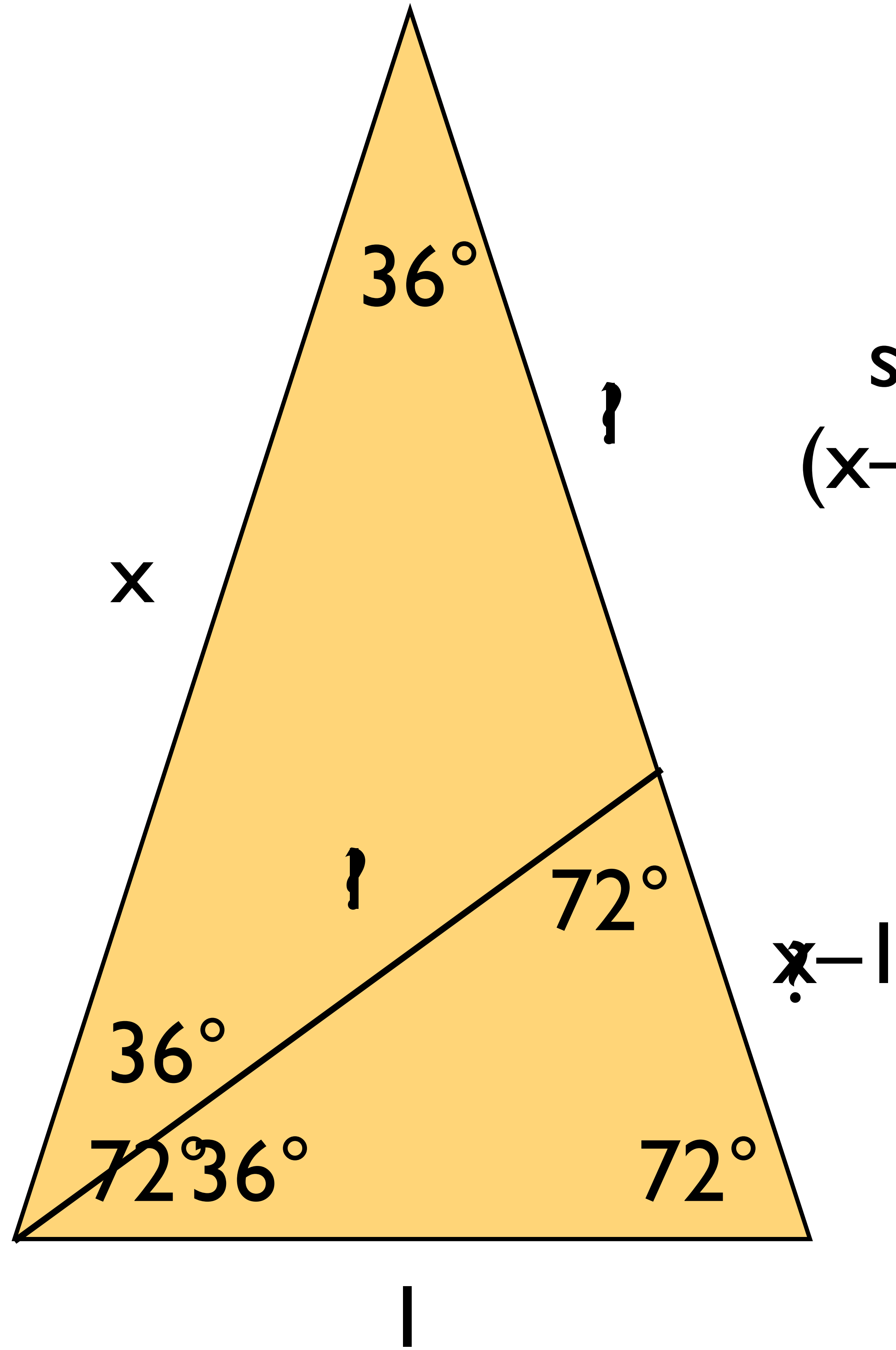






$$r = e^{-kt}$$

72°-36°-72° Triangle



ratio of short/long

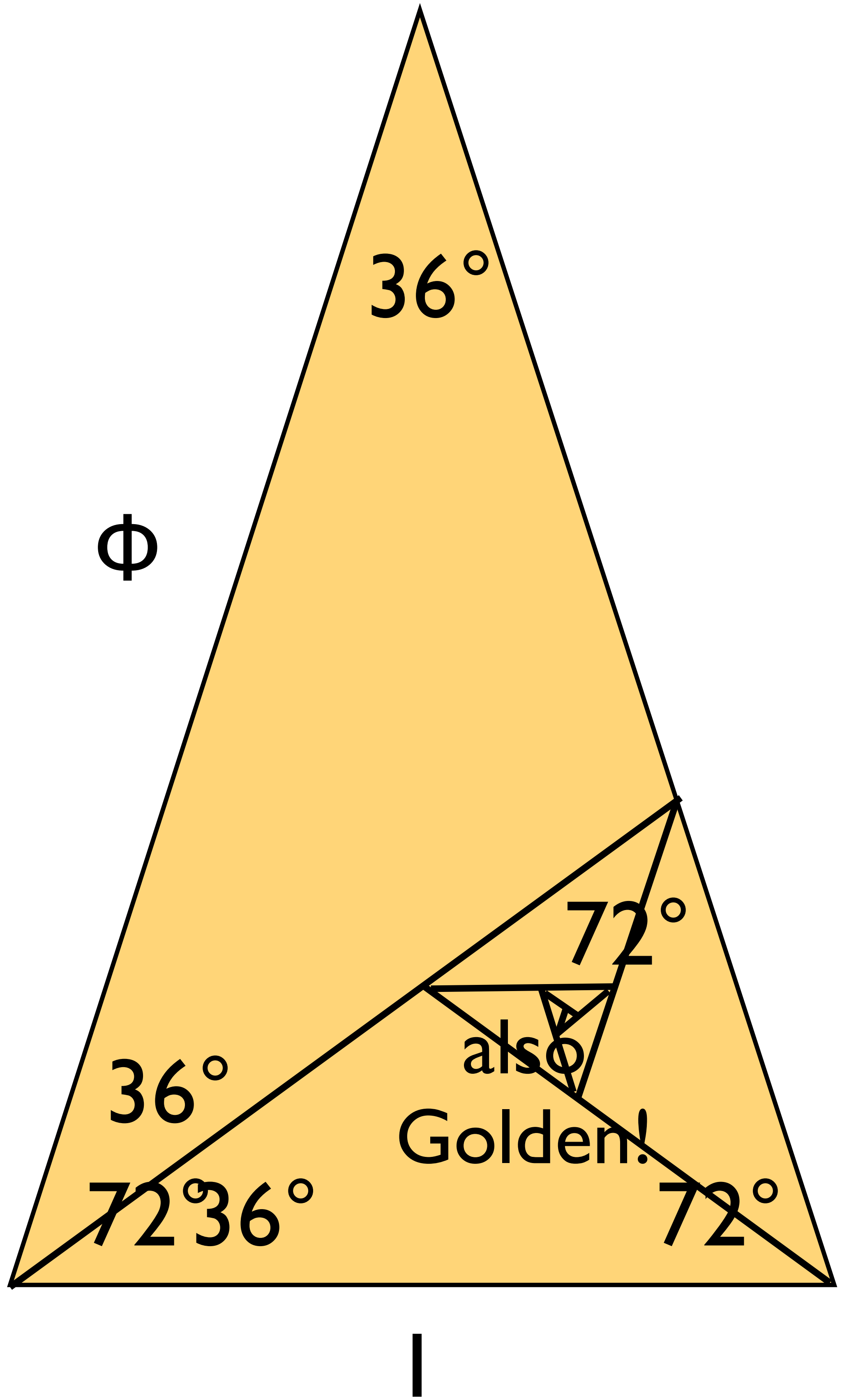
small: large:

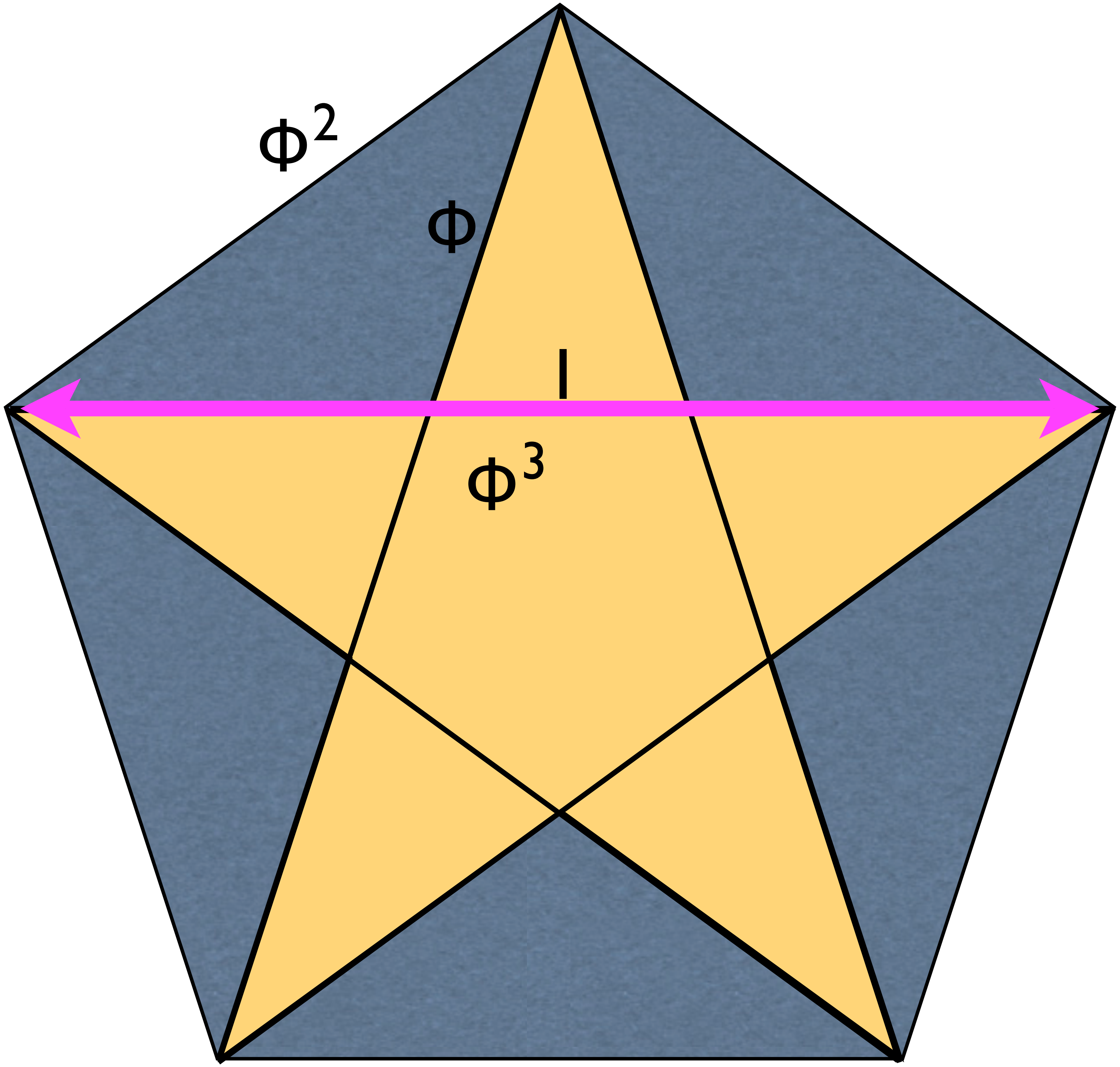
$$(x-1) / 1 = 1/x$$

$$x-1 = 1/x$$

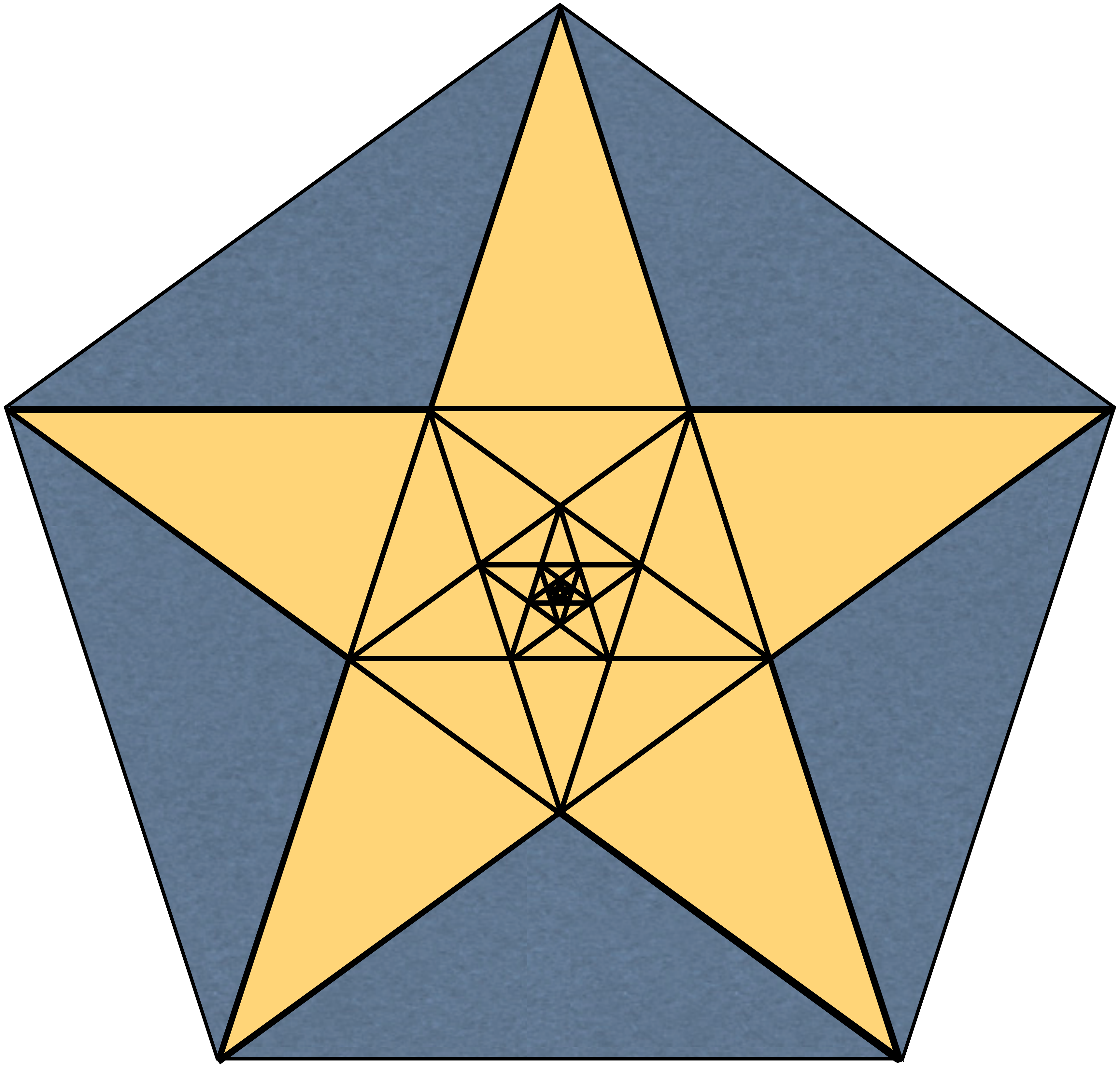
$$x = \Phi$$

Golden Triangle

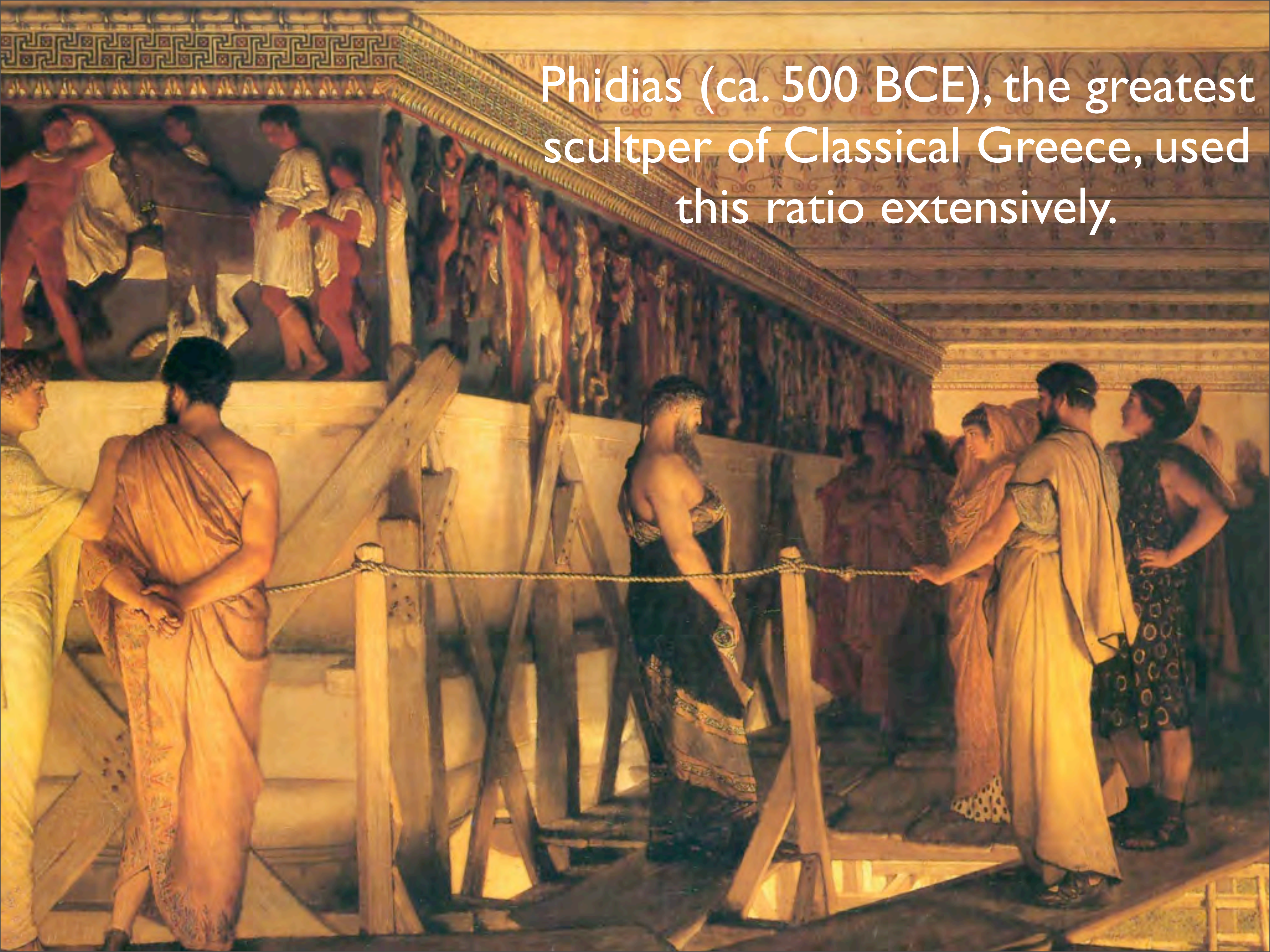


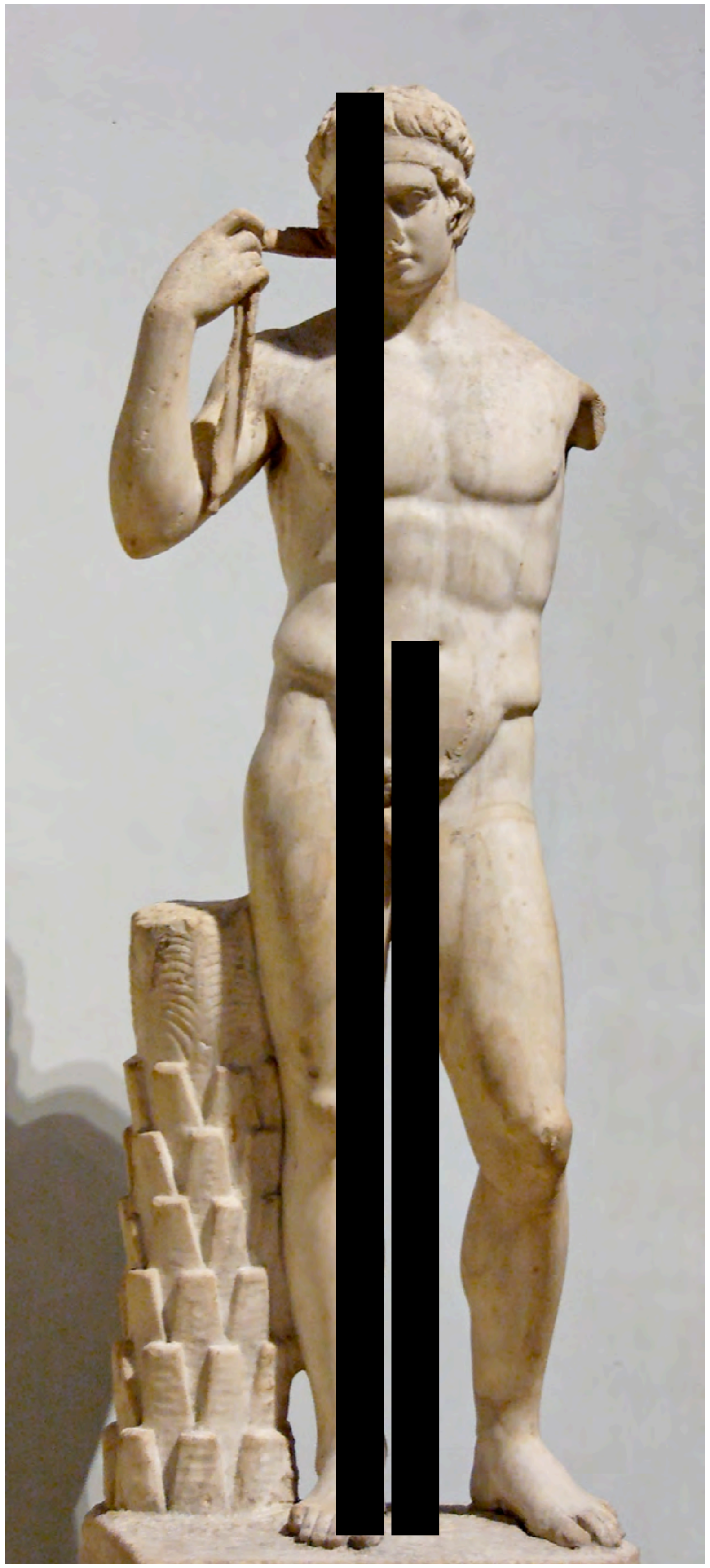


Φ2
Φ
1
Φ3



Phidias (ca. 500 BCE), the greatest scultper of Classical Greece, used this ratio extensively.





It is reported than in many of Phidias’ statues:

$$\frac{\text{Height of Figure}}{\text{Height of Belly Button}} = \Phi$$

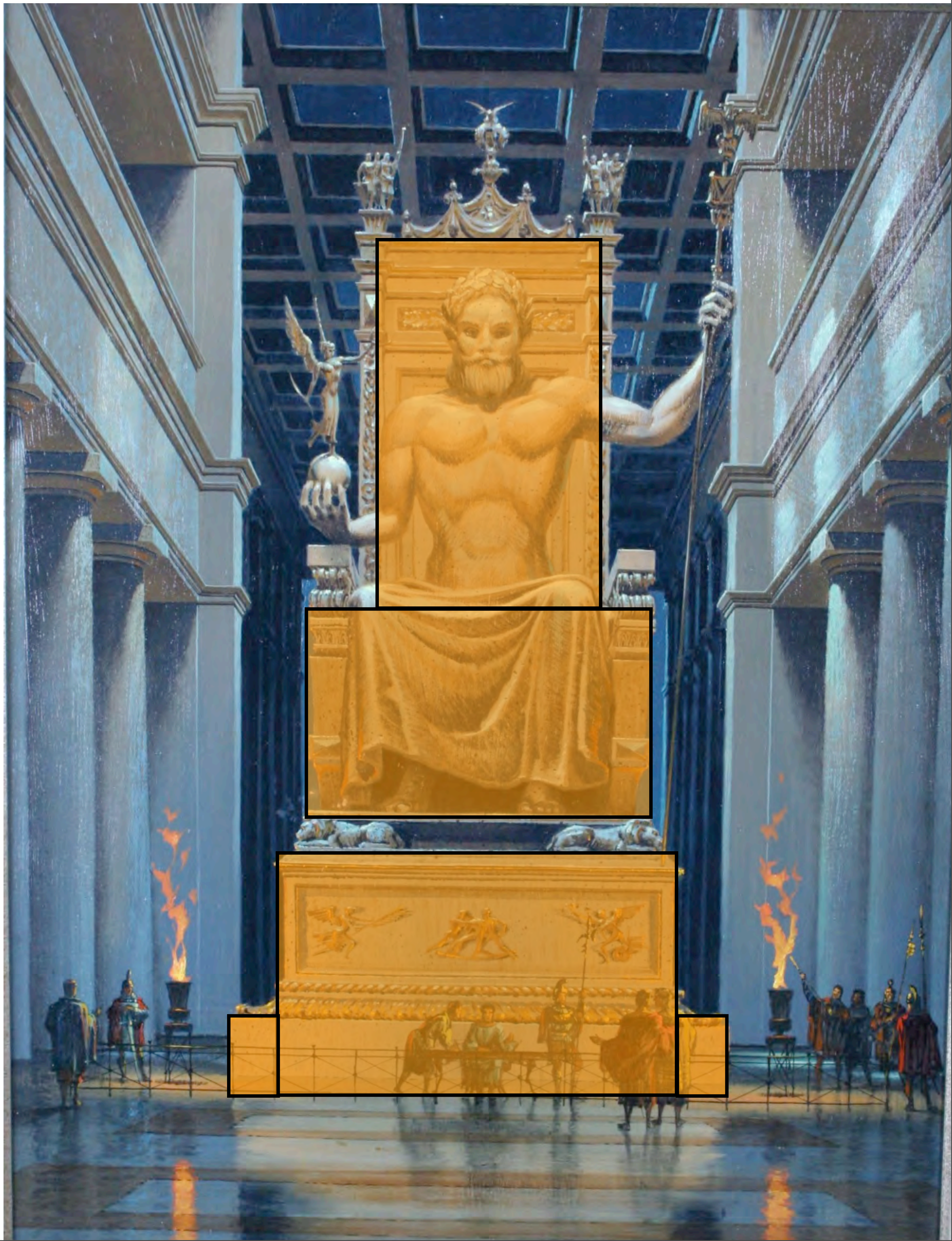
$$= 1.618...$$

It was thought that this was the most beautiful position for one’s belly button.

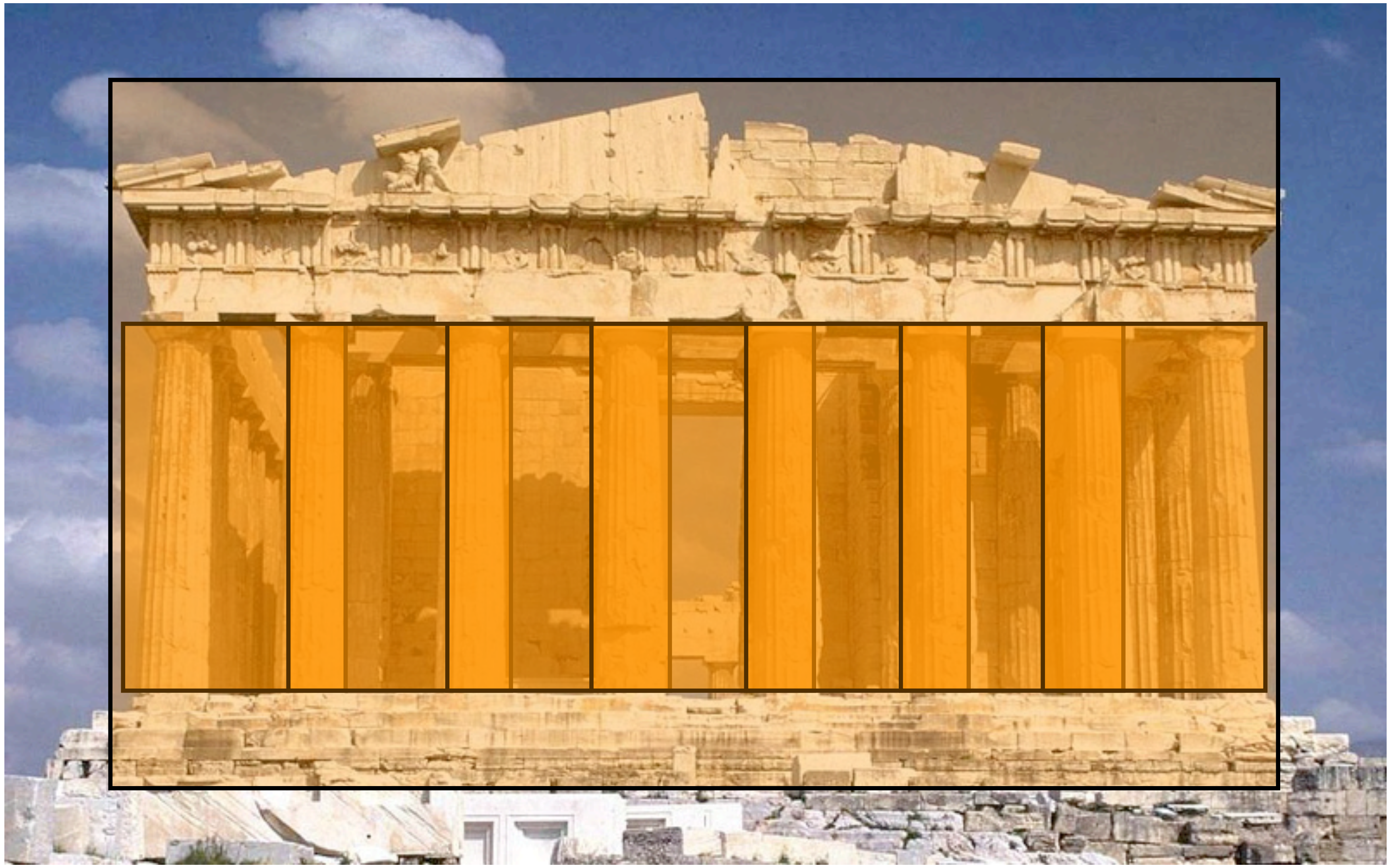
It is also said that
his work contained
many golden rectangles.

$$\frac{\text{length}}{\text{width}} = \Phi$$

Statue of Zeus, one of the 7 wonders of the ancient world, built by Phidias.



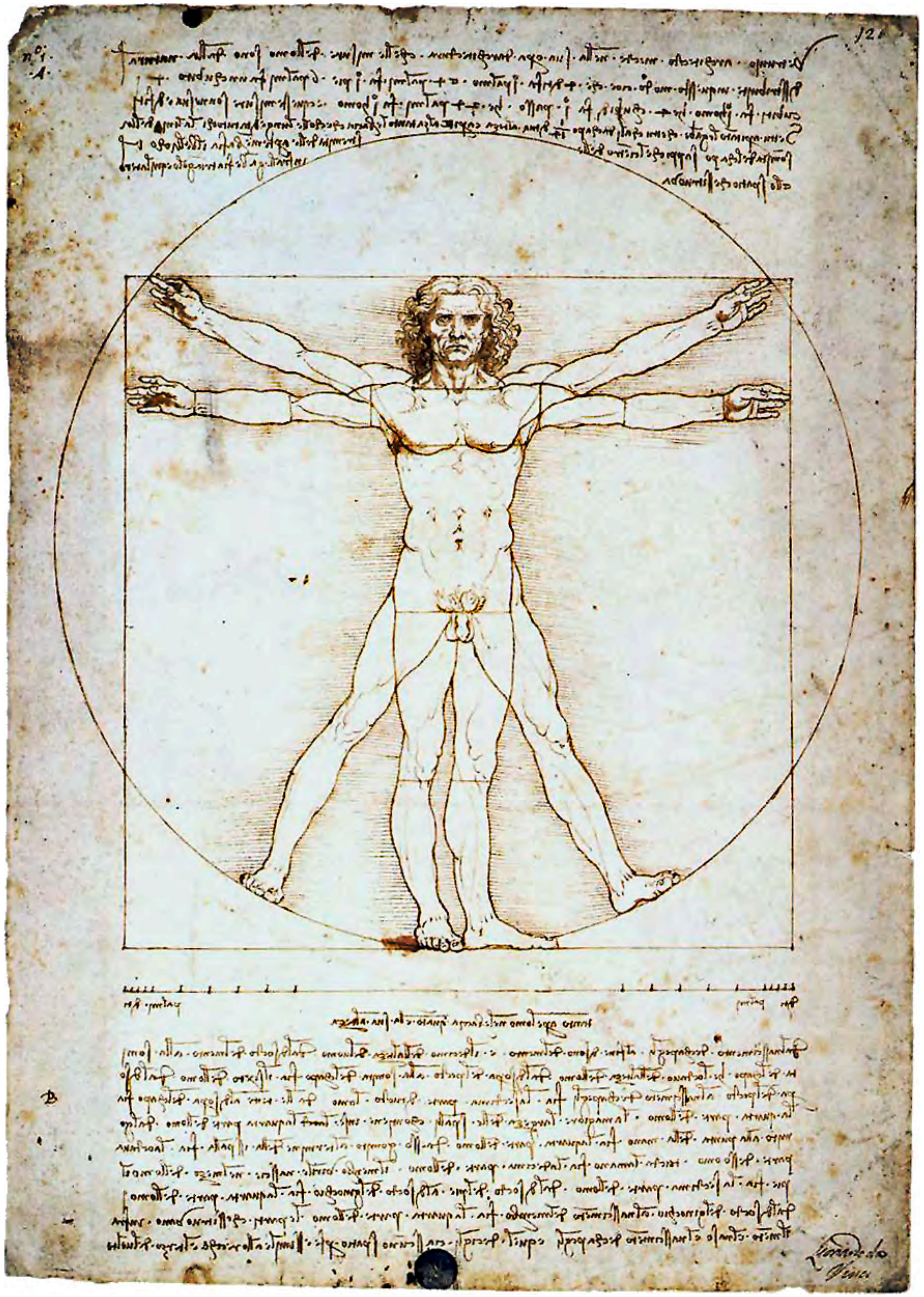
Phidias' work once filled the Parthenon,
which seems to contain many golden rectangles.

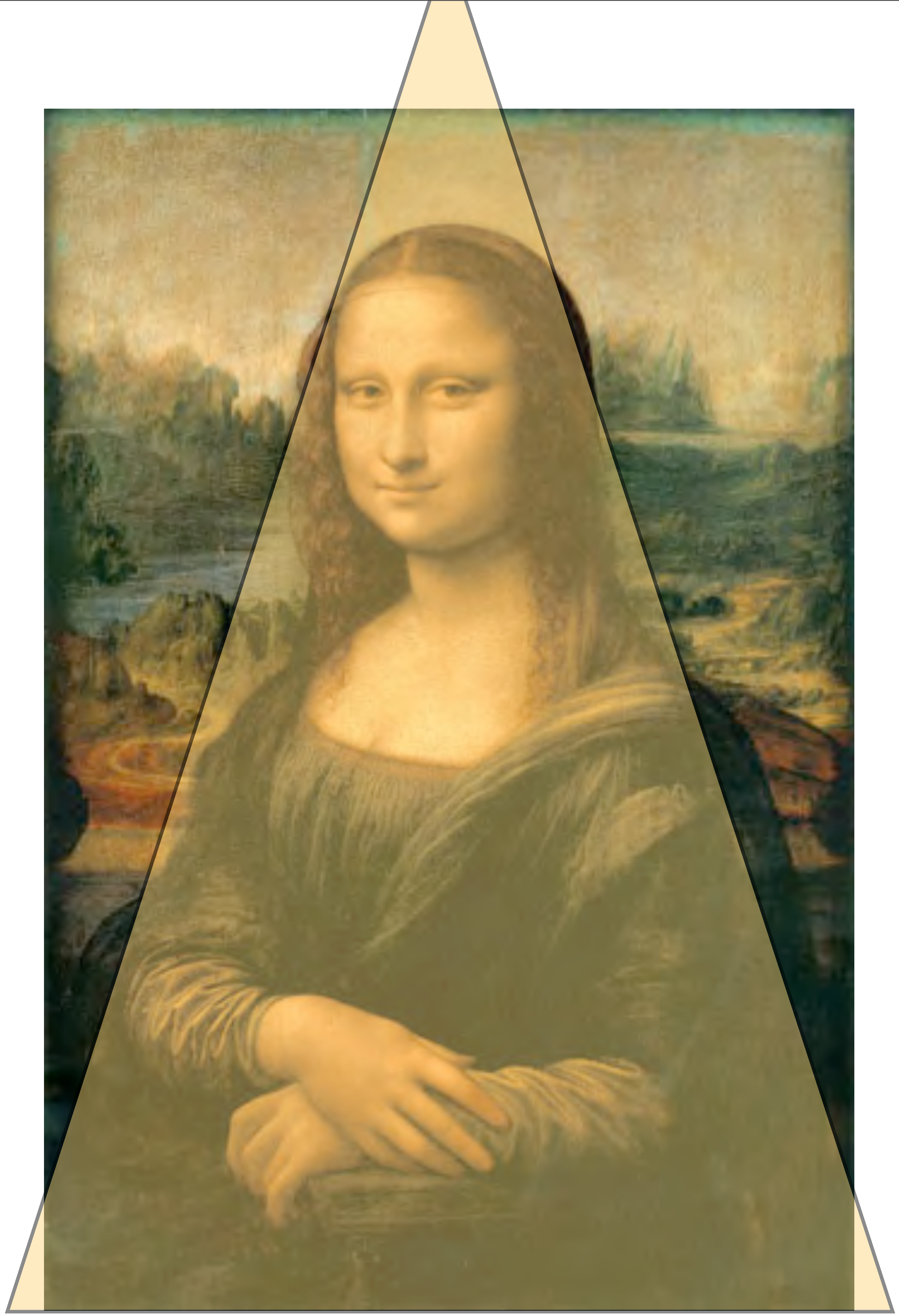


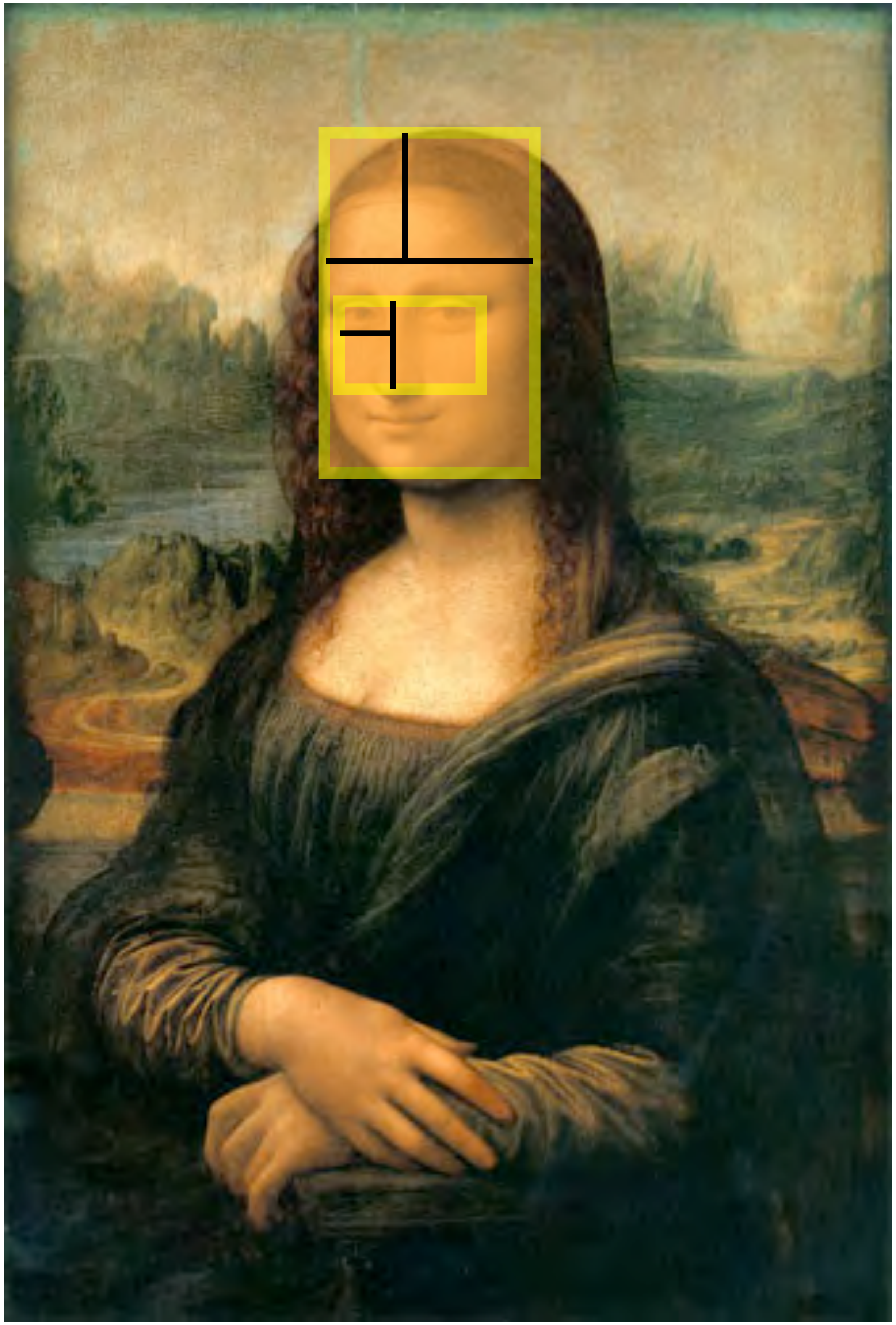
Φ is named for Phideas!

Φιδίας

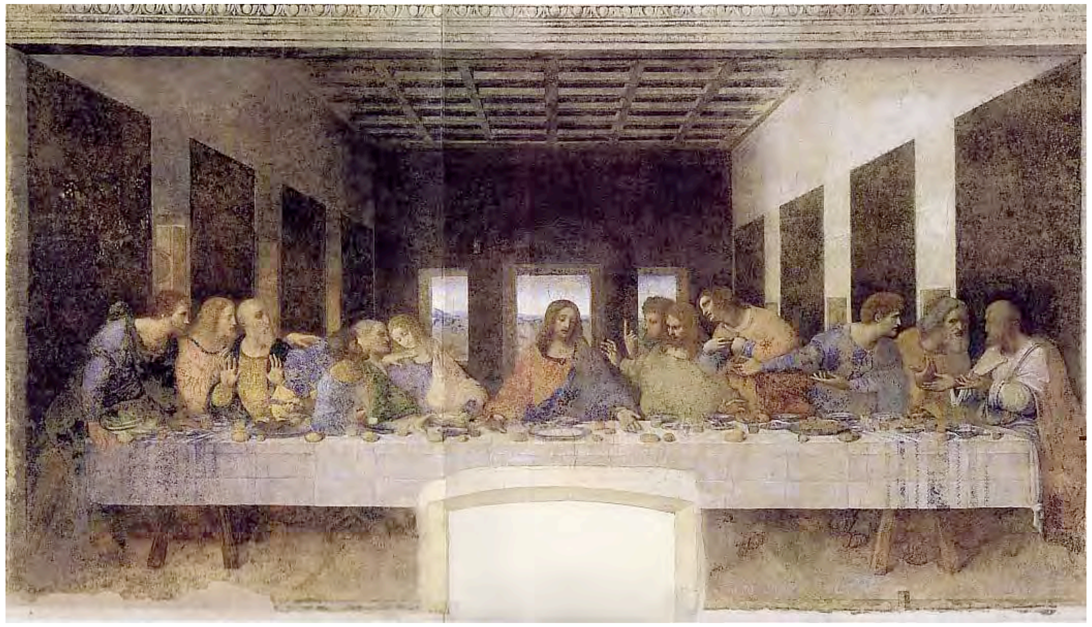
Throughout the ages, many artists have used the golden ratio, most famously Italian Renaissance man Leonardo DaVinci (ca. 1500)







Many famous artists have intentionally used the golden ratio in their art.

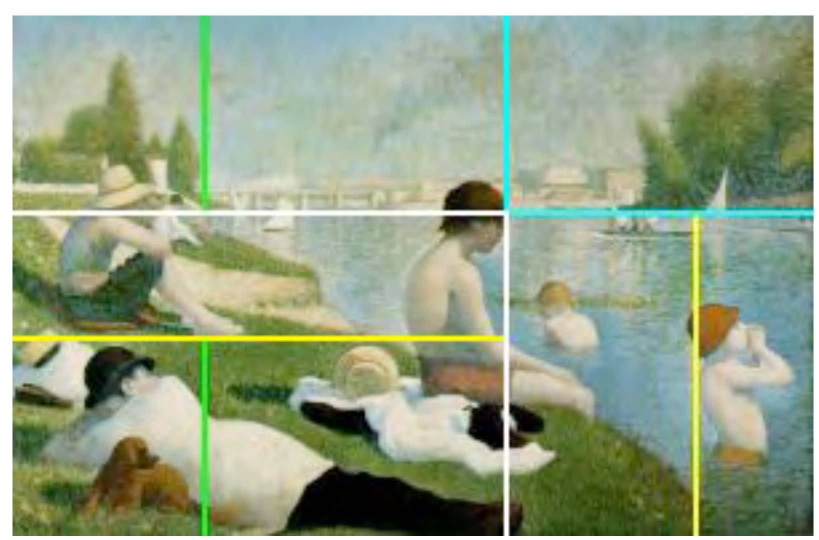


DaVinci

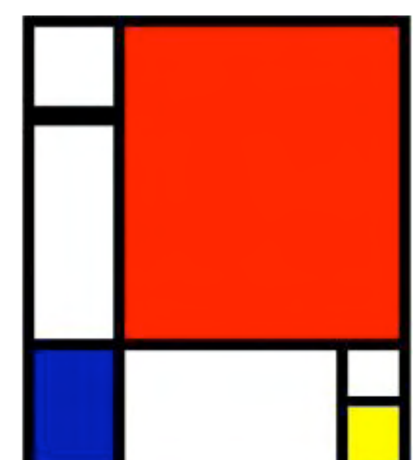
Michaelangelo

Raphael

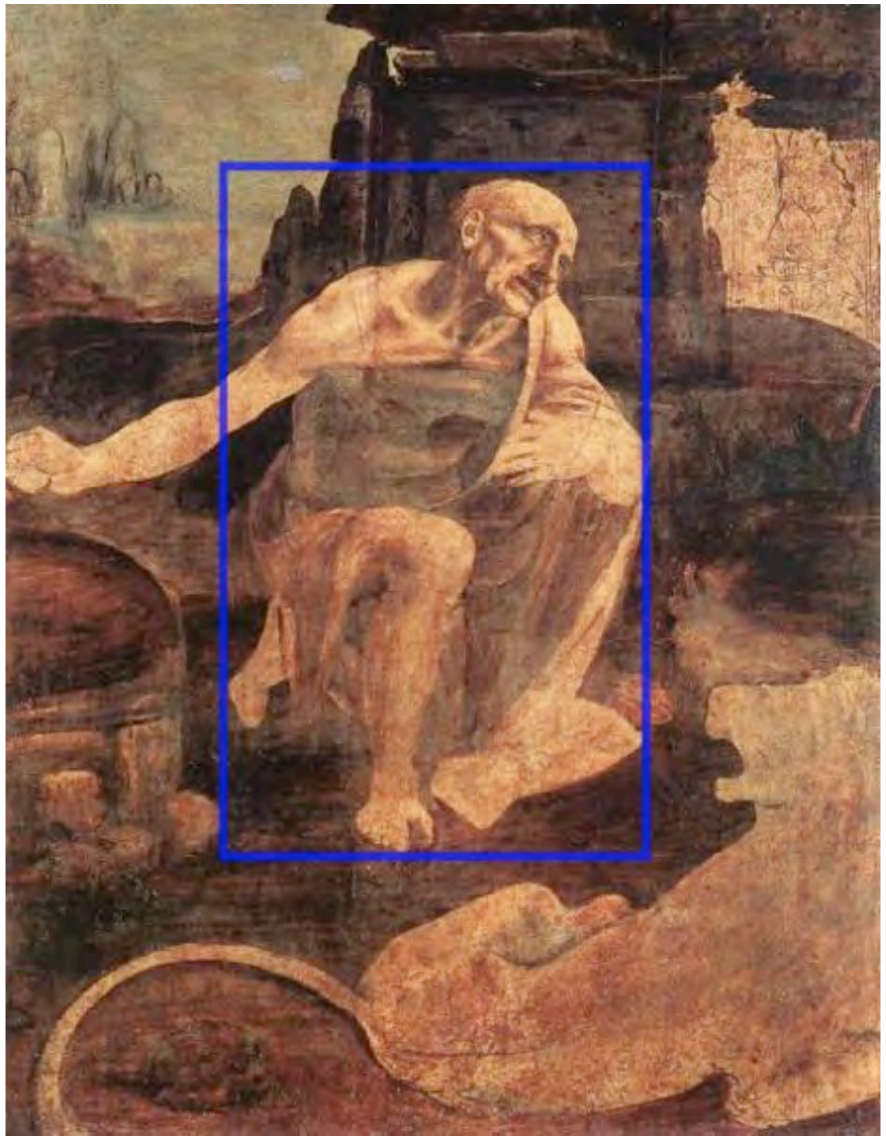
Dali



Seurat



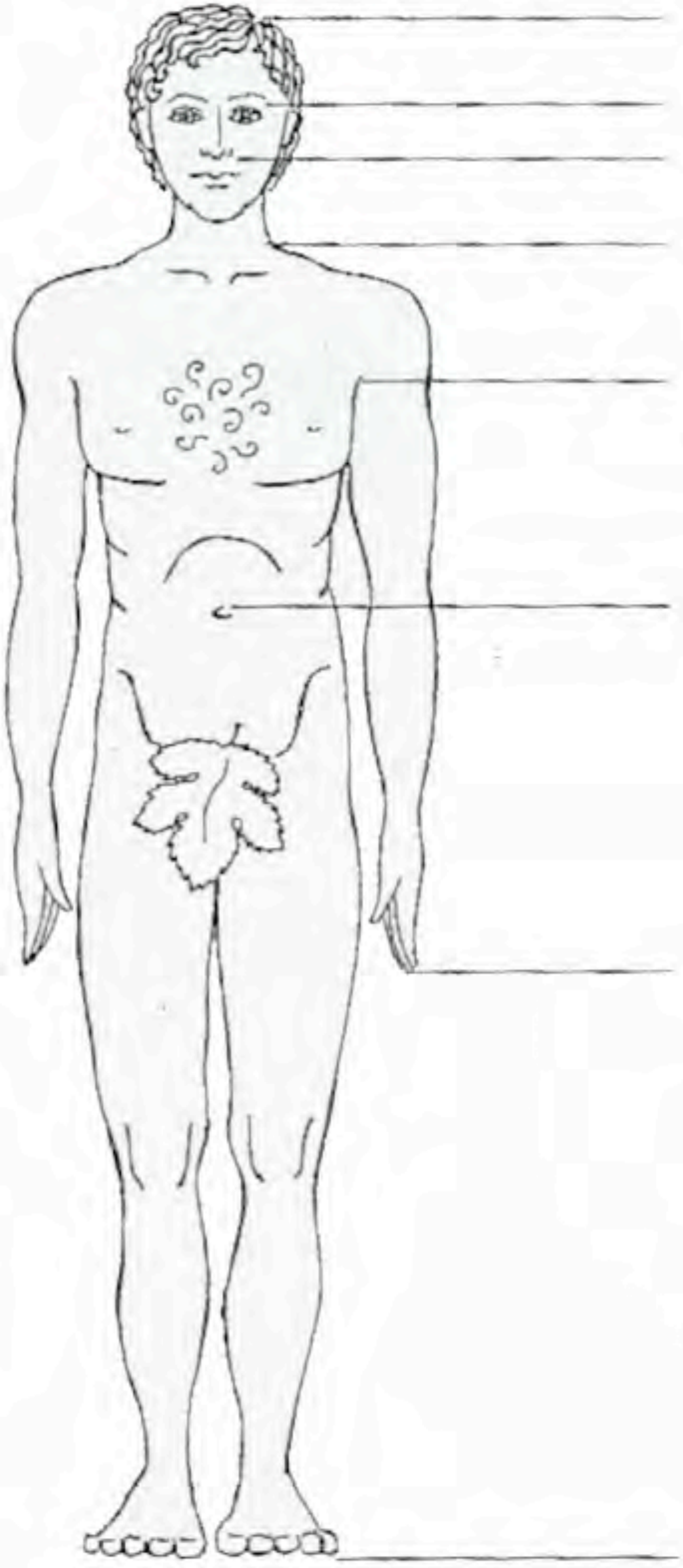
Mondrian



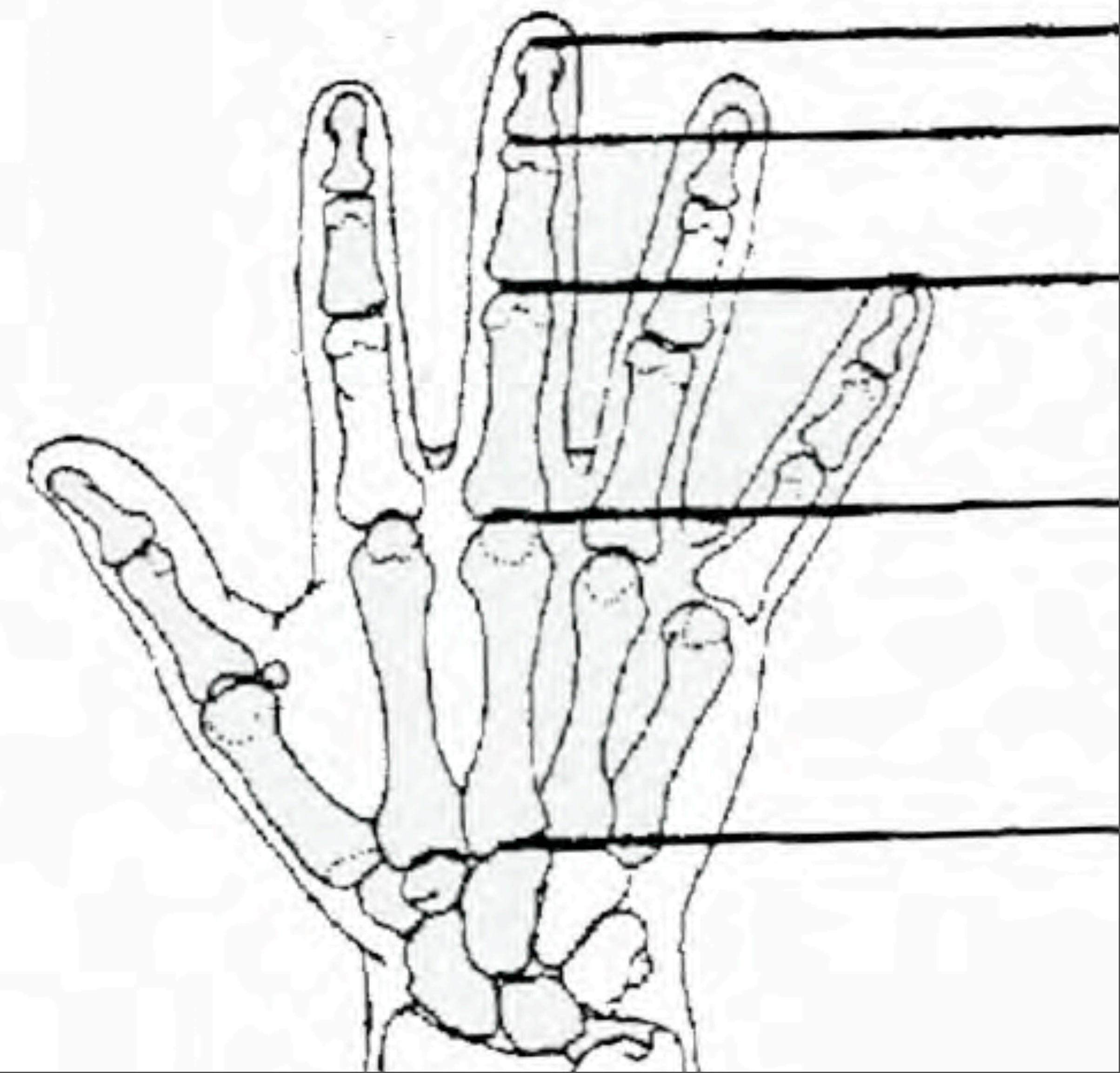
But... are these shapes really more beautiful?

Many psychological studies have been done over the past 200 years...

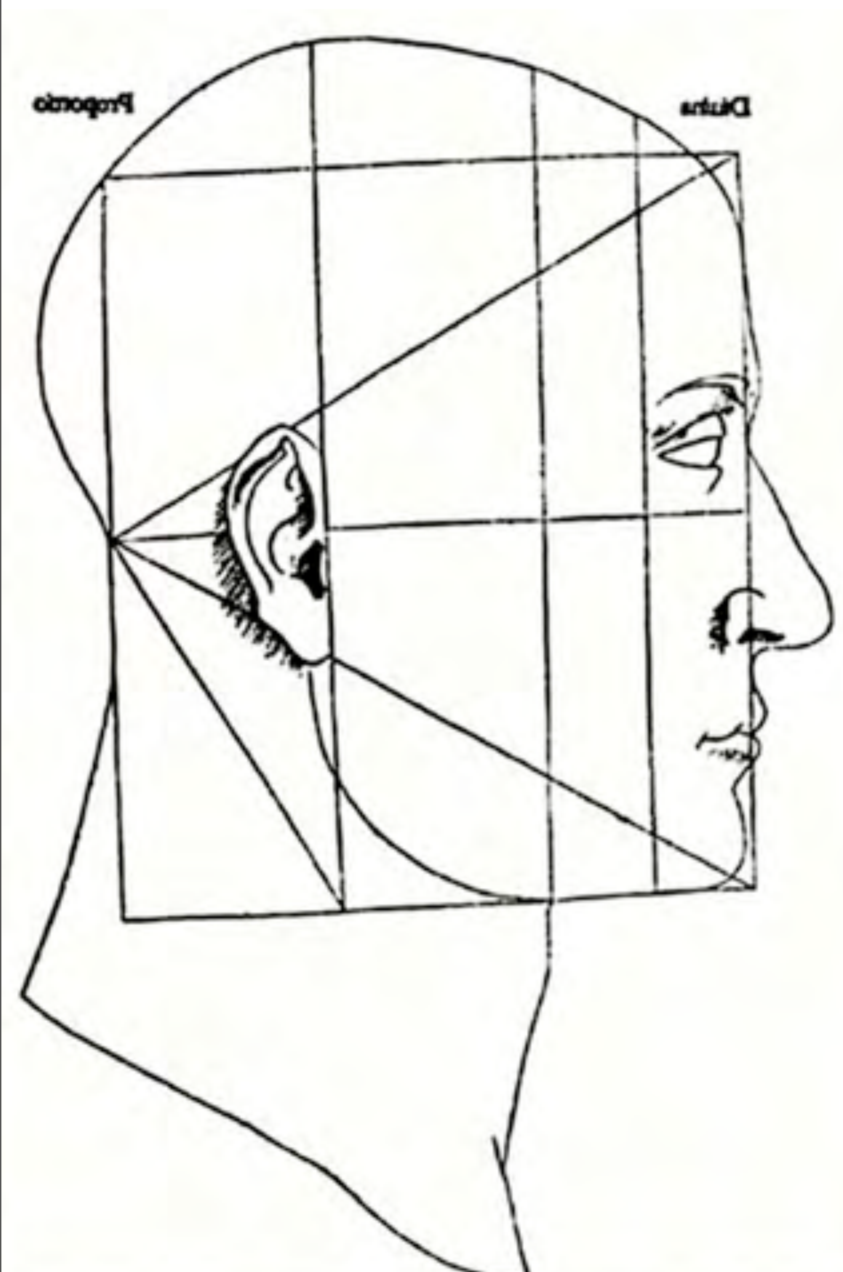
and there is no conclusive evidence that people prefer this ratio.



There are many claims that the ratio can be found all over the body...



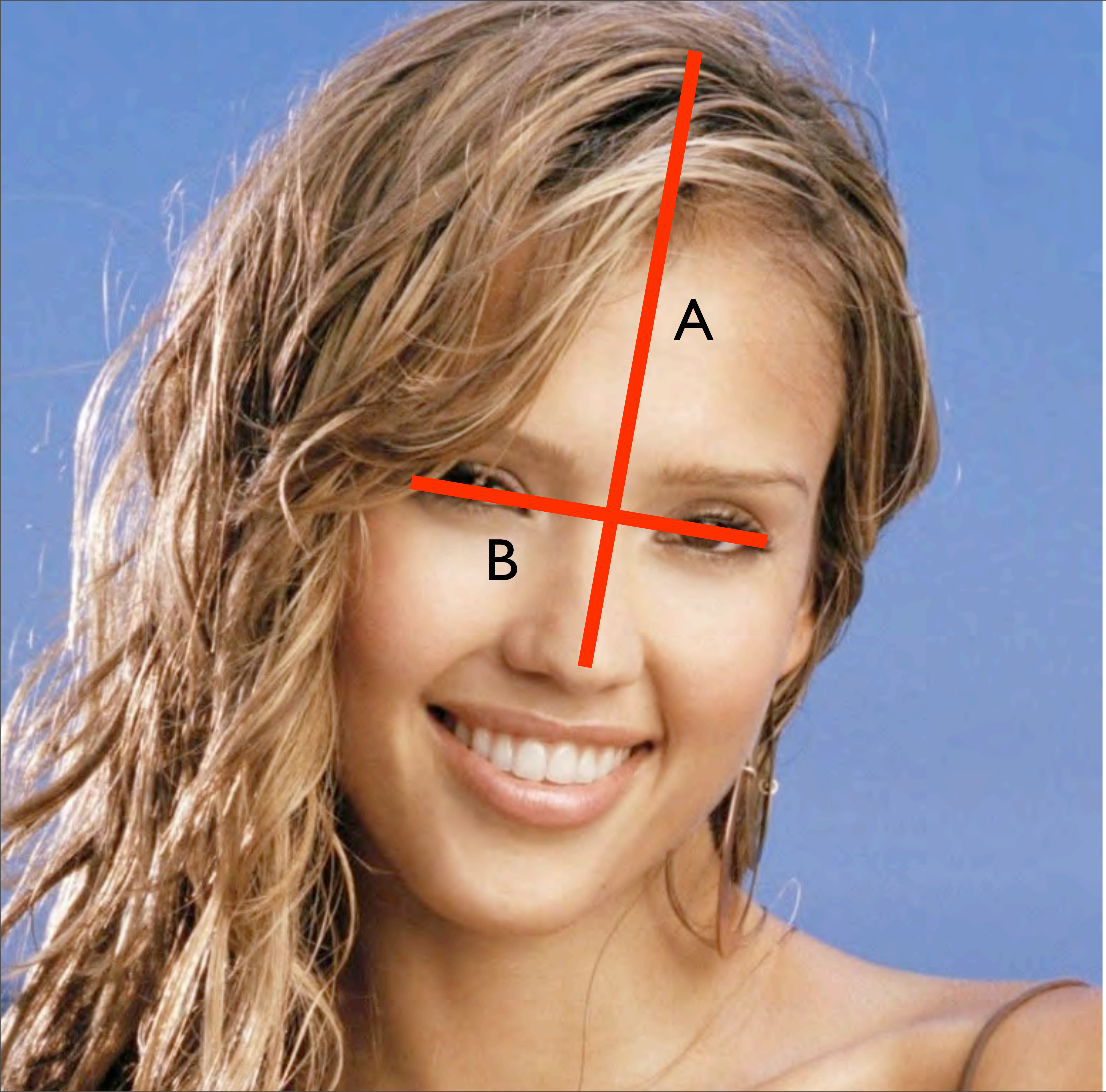
... and that the most beautiful faces conform closely to the golden ratio.



(as if we don't have enough reasons to feel bad about how we look!)

Is it true?

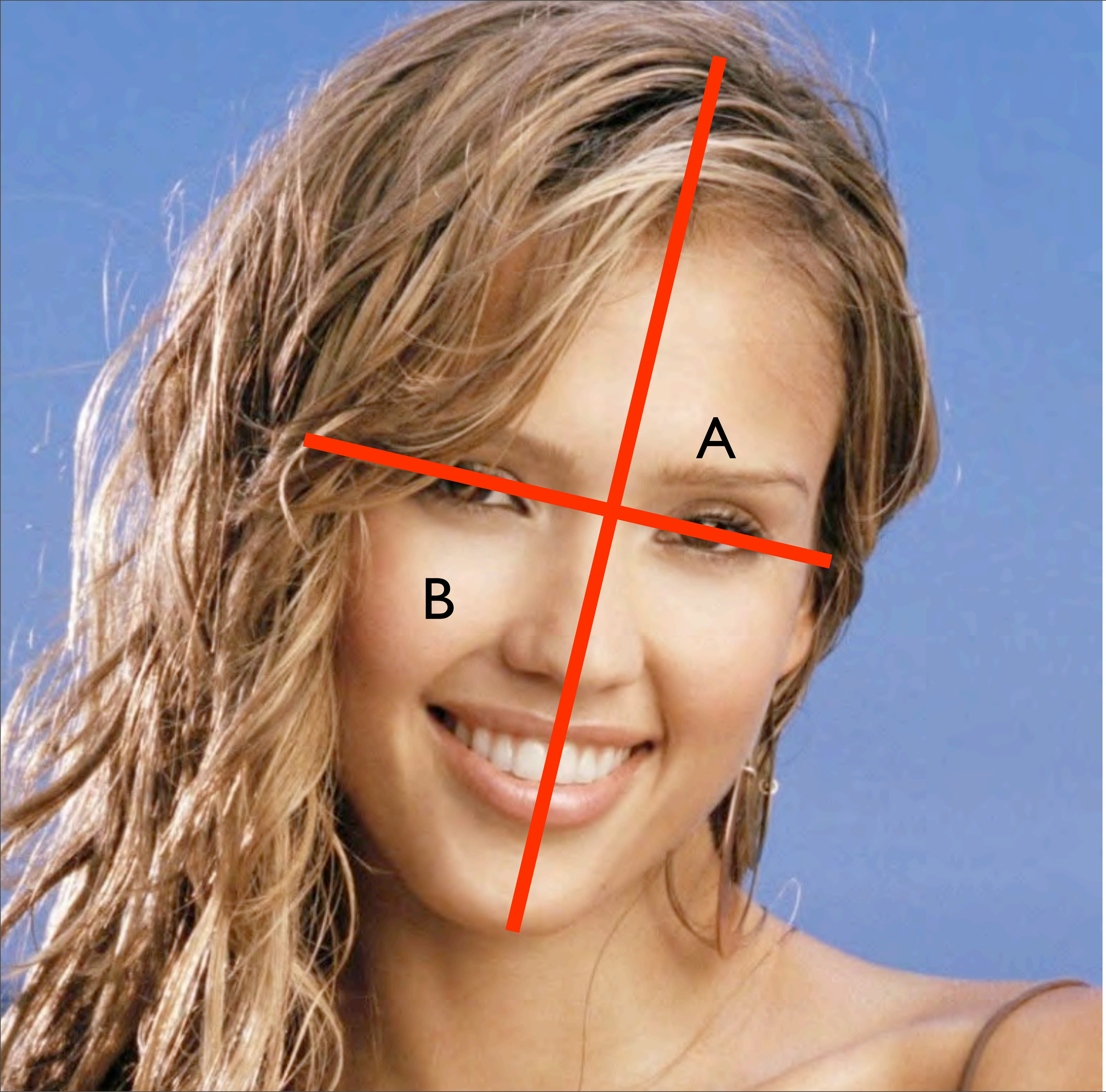
Is this ratio part of the way our bodies are assembled?



$$\frac{\text{Distance A}}{\text{Distance B}} = 1.6$$

Golden Ratio!

WOW!

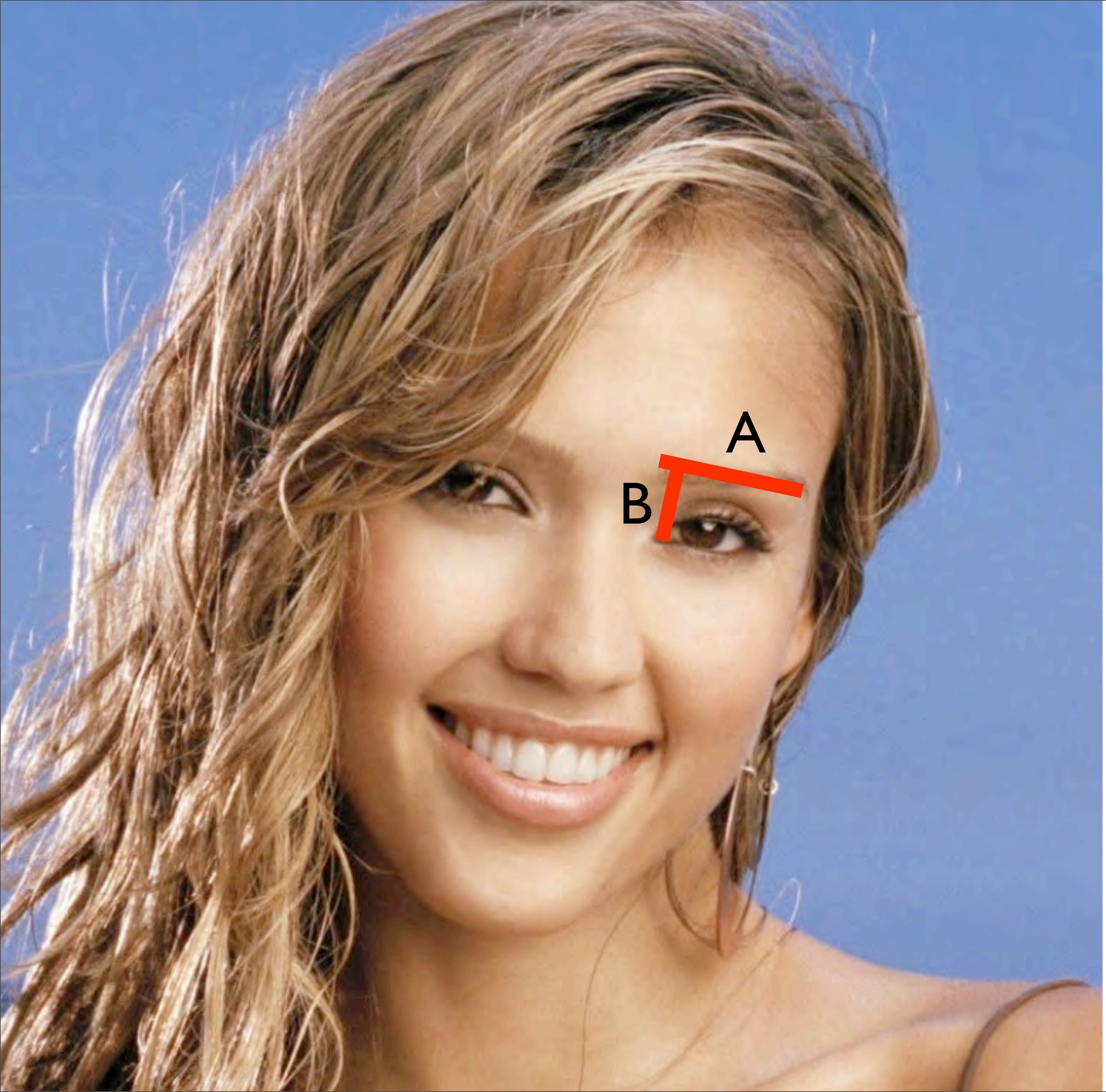


$$\frac{\text{Distance A}}{\text{Distance B}}$$

= 1.6

Golden Ratio again!

WOW!

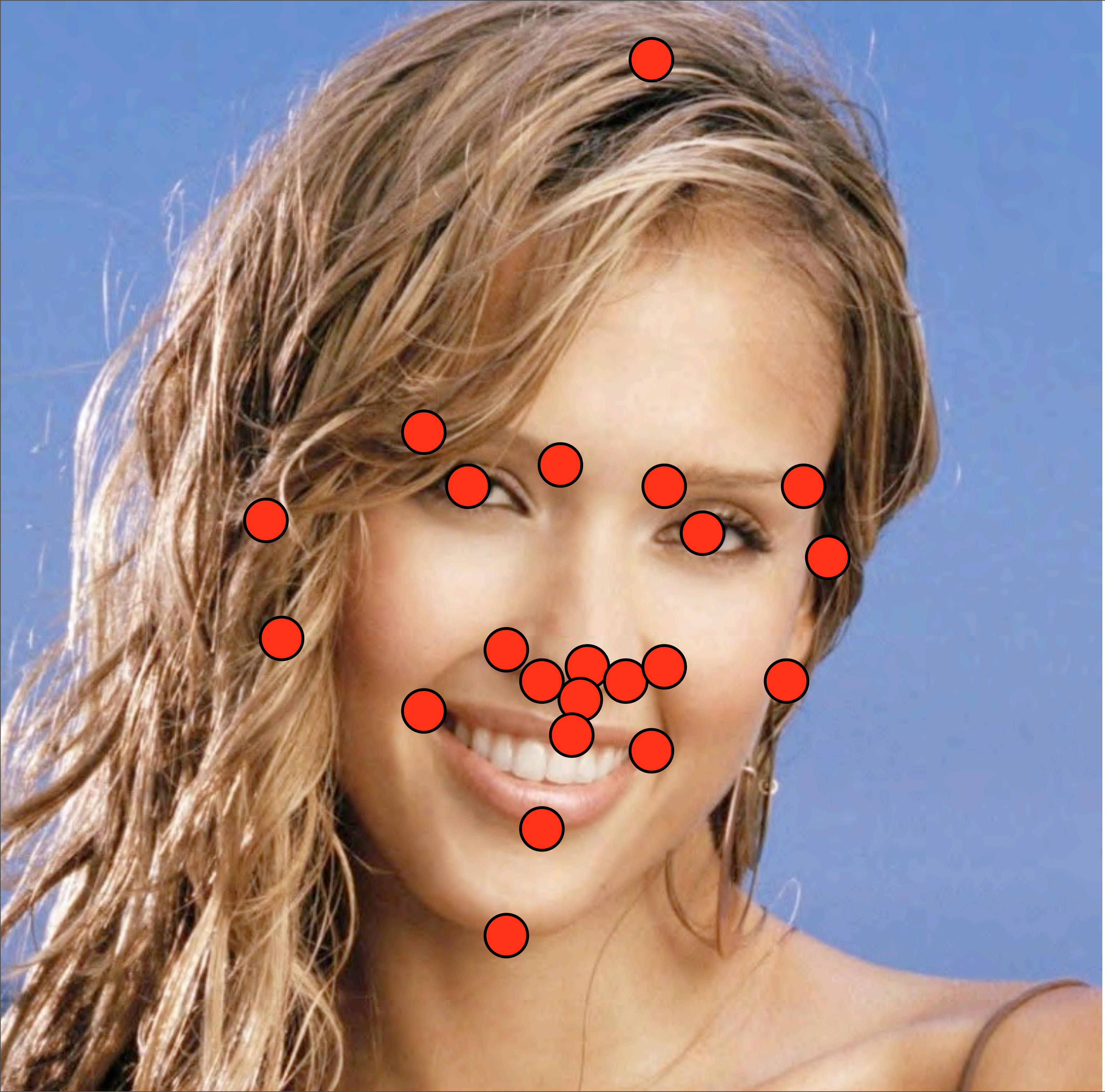


$$\frac{\text{Distance A}}{\text{Distance B}}$$

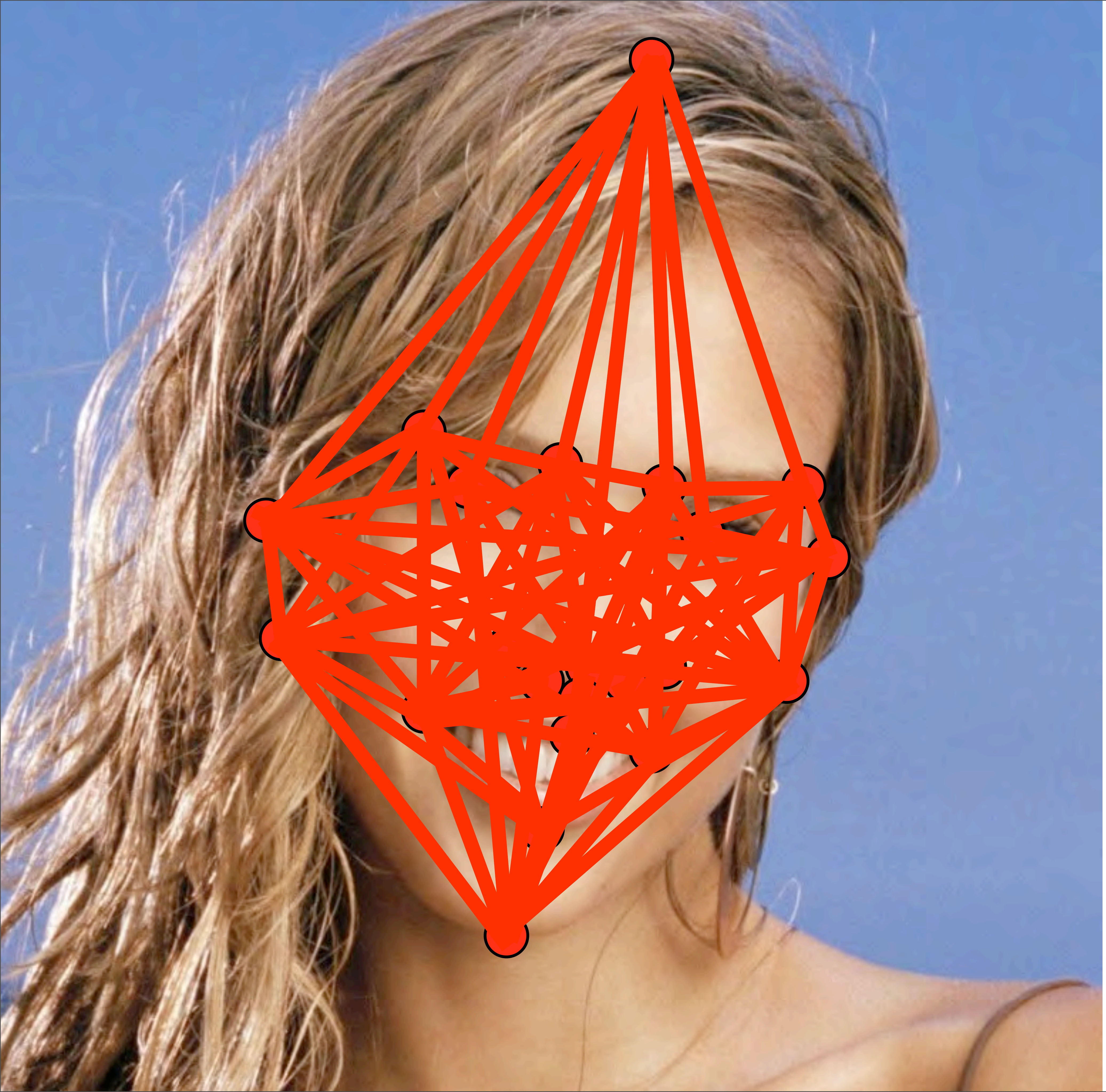
= 2

(never mind, ignore that...)

Wait a minute...

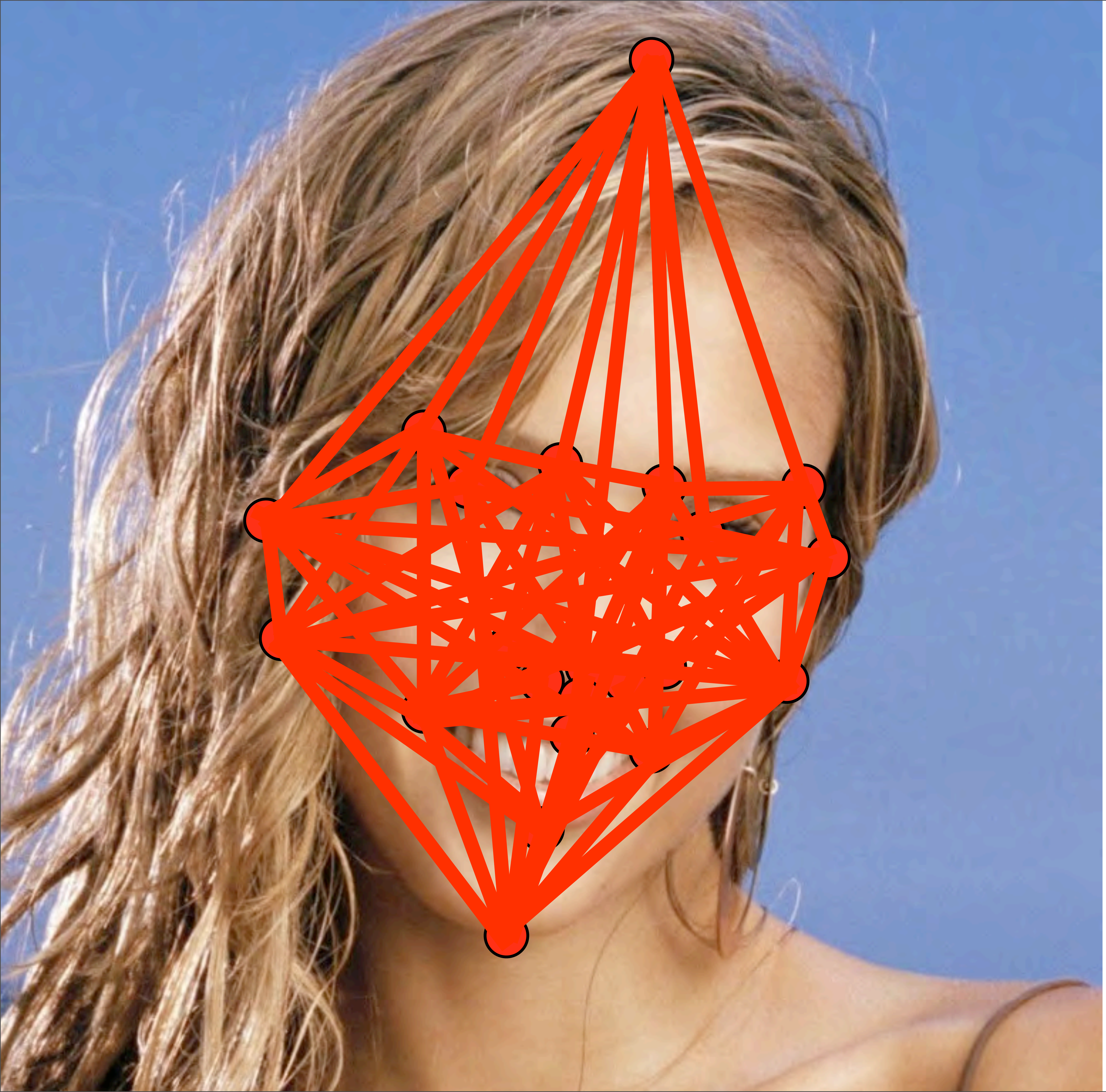


22 points



22 points

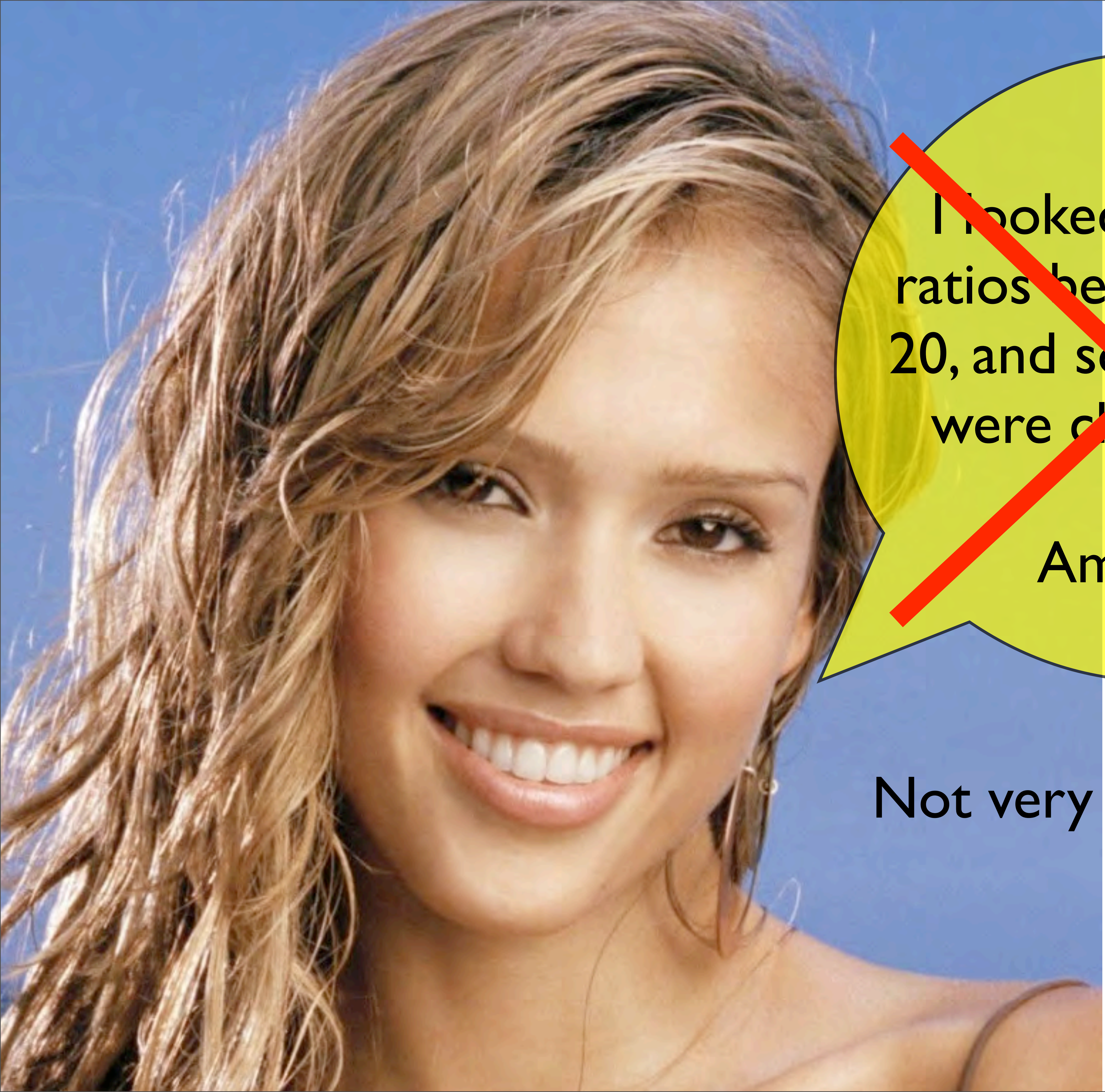
lengths?
ratios
(≠ 1)?



22 points

22 x 21÷2 =
231 lengths

231 x 230 =
53,130
ratios



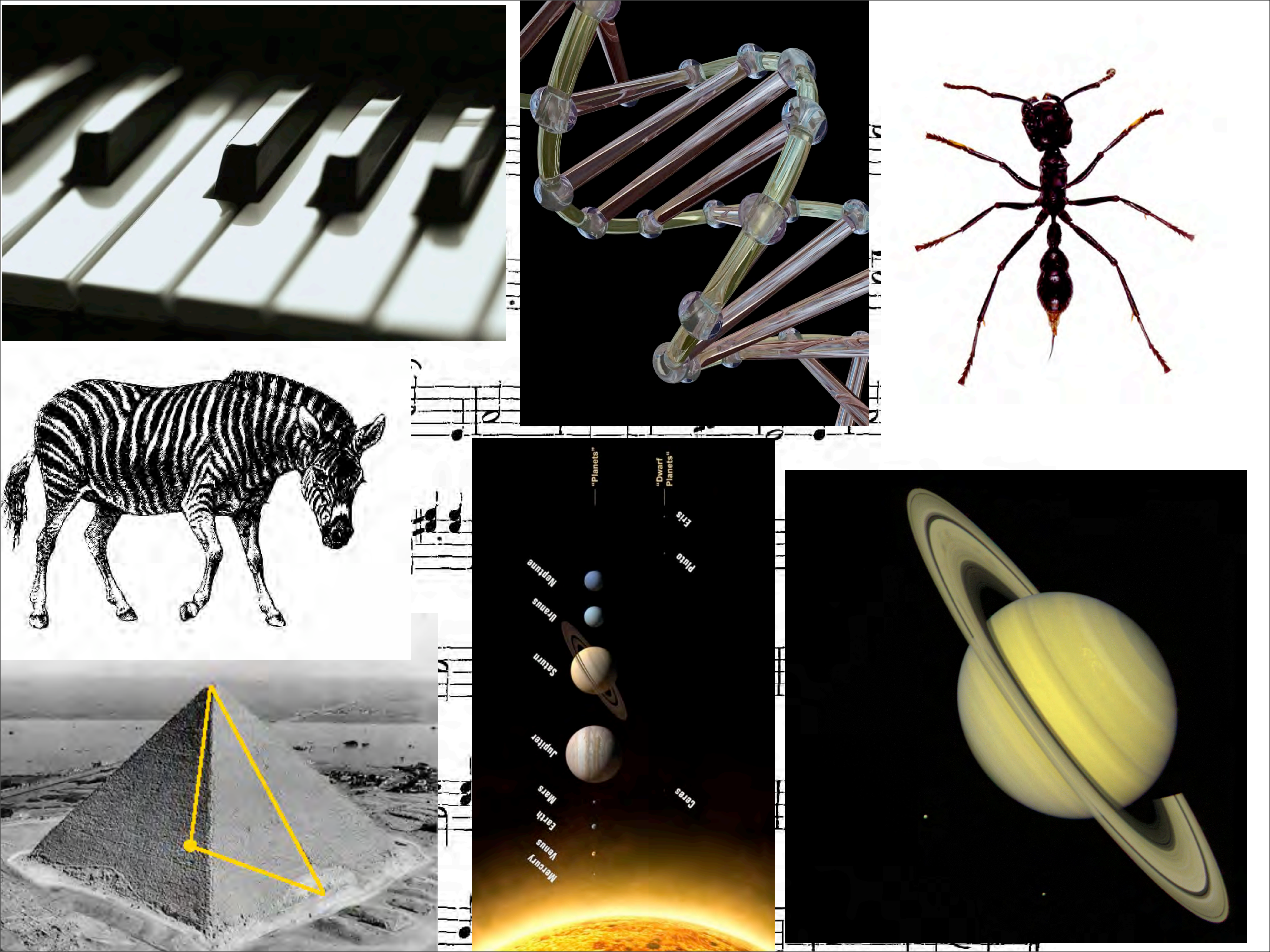
~~I looked at 50,000 ratios between 0 and 20, and some of them were close to 1.6!~~

~~Amazing?~~

Not very amazing.

There are many other claims about the golden ratio that are

- coincidences
- wishful thinking
- bad approximations


"Planets"
"Dwarf Planets"
Eris
Pluto
Neptune
Uranus
Saturn
Jupiter
Ceres
Mars
Earth
Venus
Mercury




If you’re looking for it, you can find it!

You can also find any other number you want.




Be wary! It is not enough to find a few measurements that are "close" to the ratio.

Ask: Is there a reason?

The Golden Ratio DOES appear in some places in nature.

The reasons are amazing.
The reasons are MATHEMATICAL.

Leonardo Pisano
(ca. 1170 – 1250)

Leonardo of Pisa

Son of Bonacci

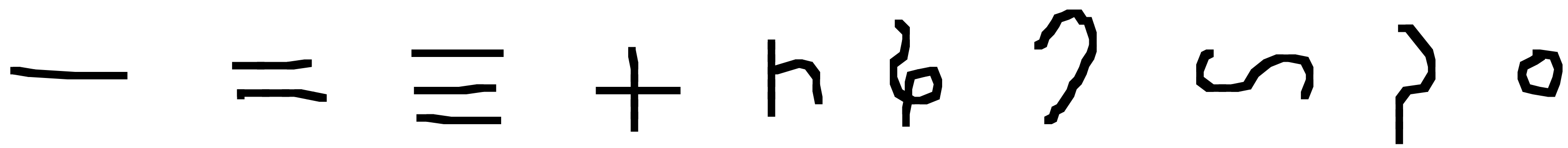
Fibonacci

Leaning Tower of Pisa
(ca. 1174 – present)

Fibonacci's Travels
(ca. 1180–1200)

Liber Abaci

“The Book of Calculating”

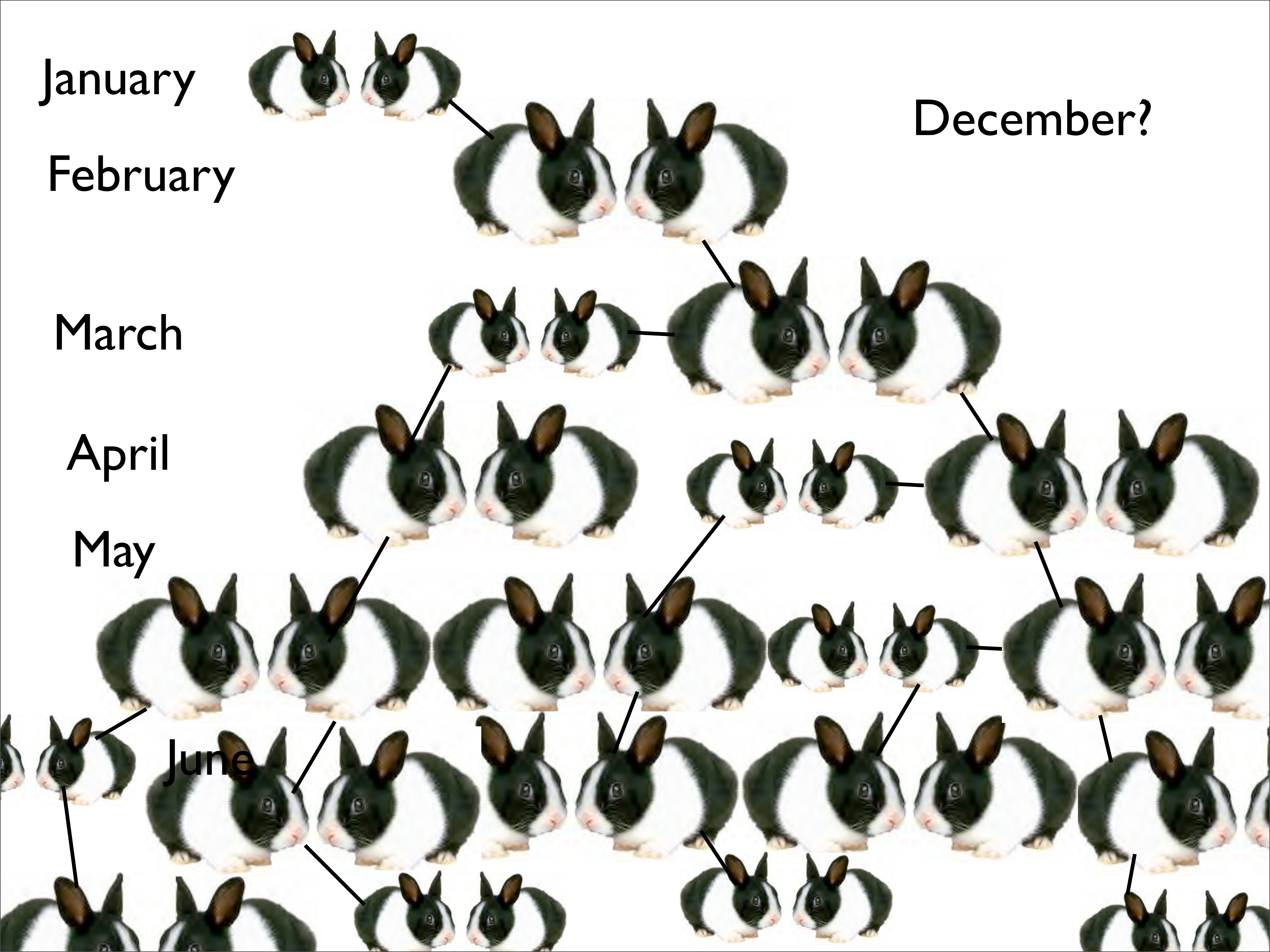


1 2 3 4 5 6 7 8 9 0

(Hindu-Arabic Numerals)

Also in the book: "The Rabbit Problem"

The solution method produces numbers that are extremely important in nature.


January
February
March
April
May
June
December?

Pairs of:	Babies	Adults
Jan	1	
Feb		1
Mar	1	1
Apr	1	2
May	2	3
Jun	3	5
Jul	5	8
Aug	8	13
Sep	13	21
Oct	21	34
Nov	34	55
Dec	55	89

total 144

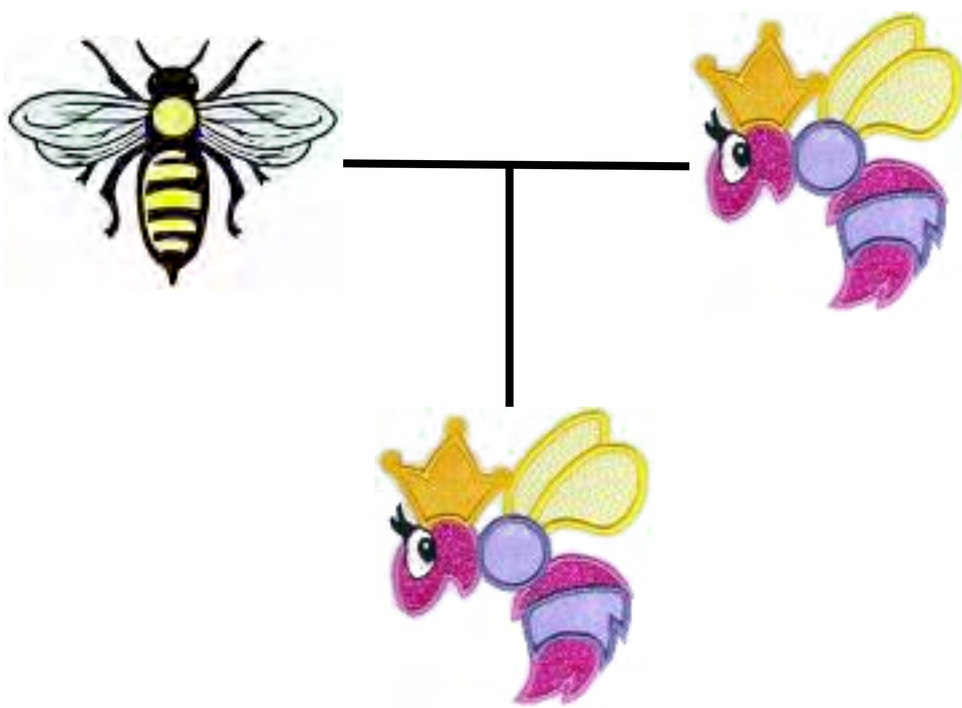
1,	1,	2,	3,	5,	8,	13,	21,	34,	55,	89,	144, 233,	377,	...

The sequence is called *the Fibonacci sequence.*

It was so named by

Edouard Lucas.
(1842–1871, Paris)

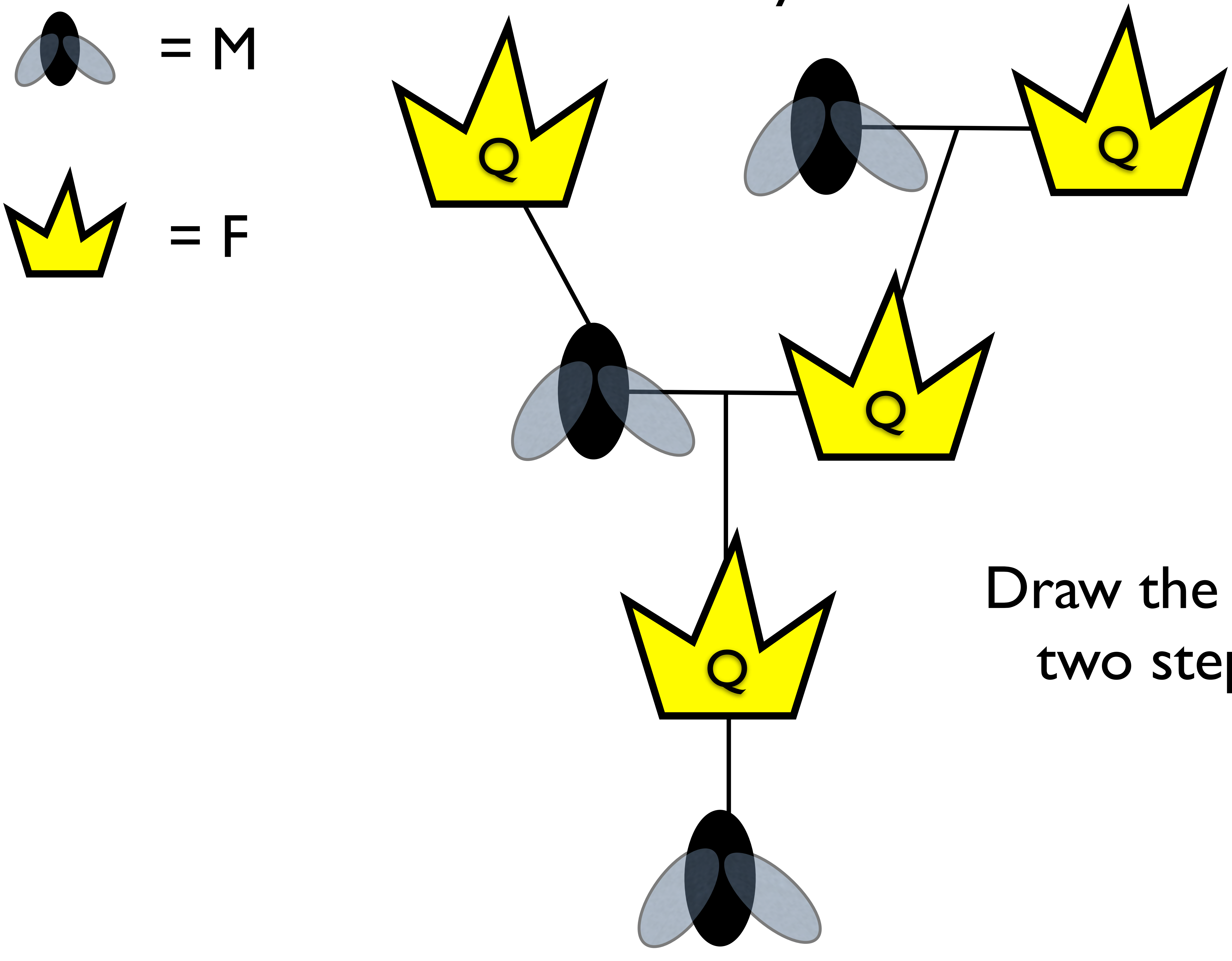
Bee Families



Fertilized Egg = Female

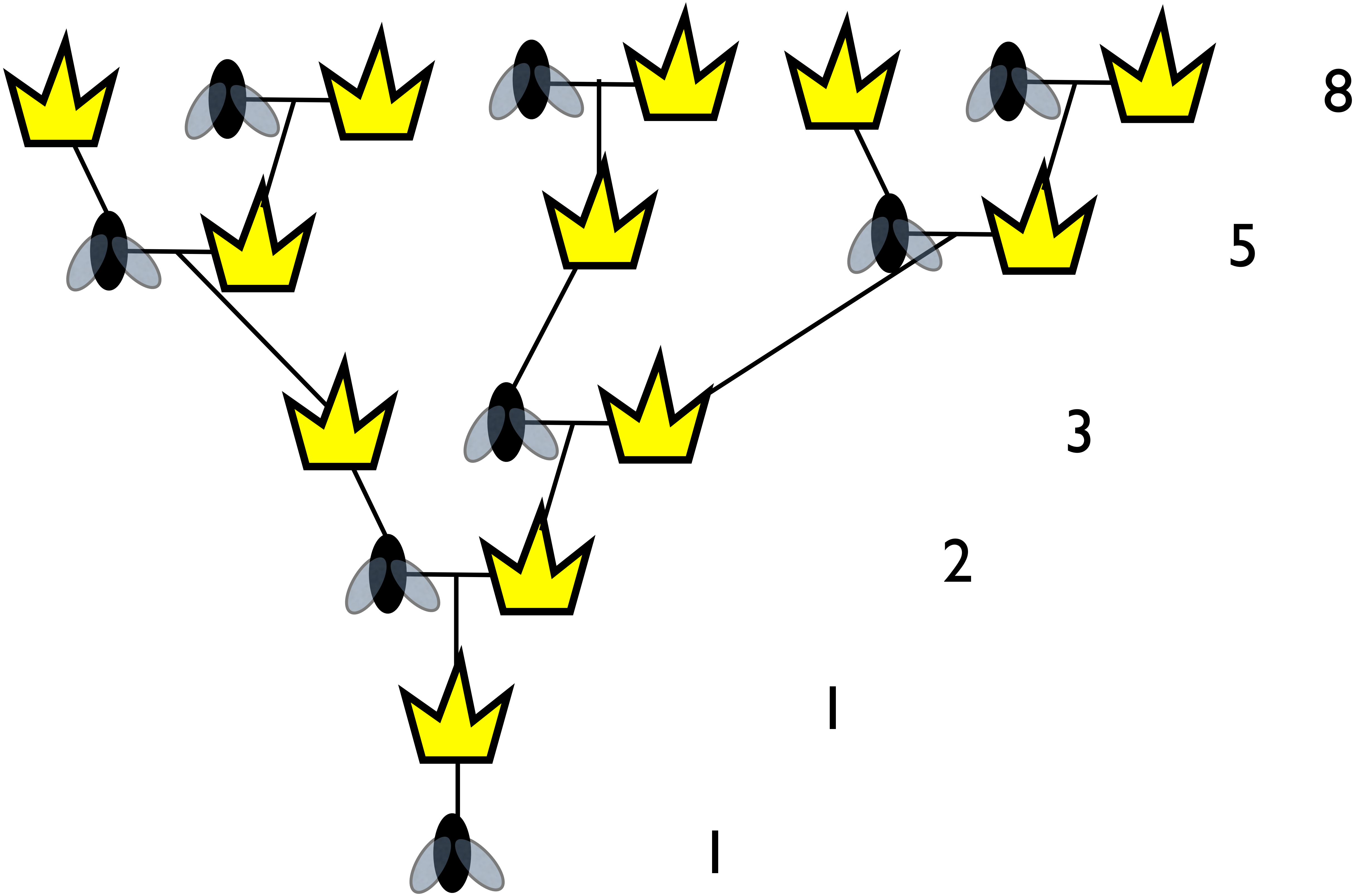
Unfertilized Egg = Male

Bee Family Tree



Draw the next two steps!

Bee Family Tree



Sneezewort


Many plants often have
Fibonacci numbers in them.

21 petals each

34 petals on
this sunflower

34 petals on this
Gerbera flower

Trilliums have 3 petals,
Buttercups and pansies have 5 petals
Delphiniums have 8 petals,
Marigolds & Black-eyes Susans & Ragwort have 13 petals,
or 21 petals
or 34 petals.

Plant Spirals

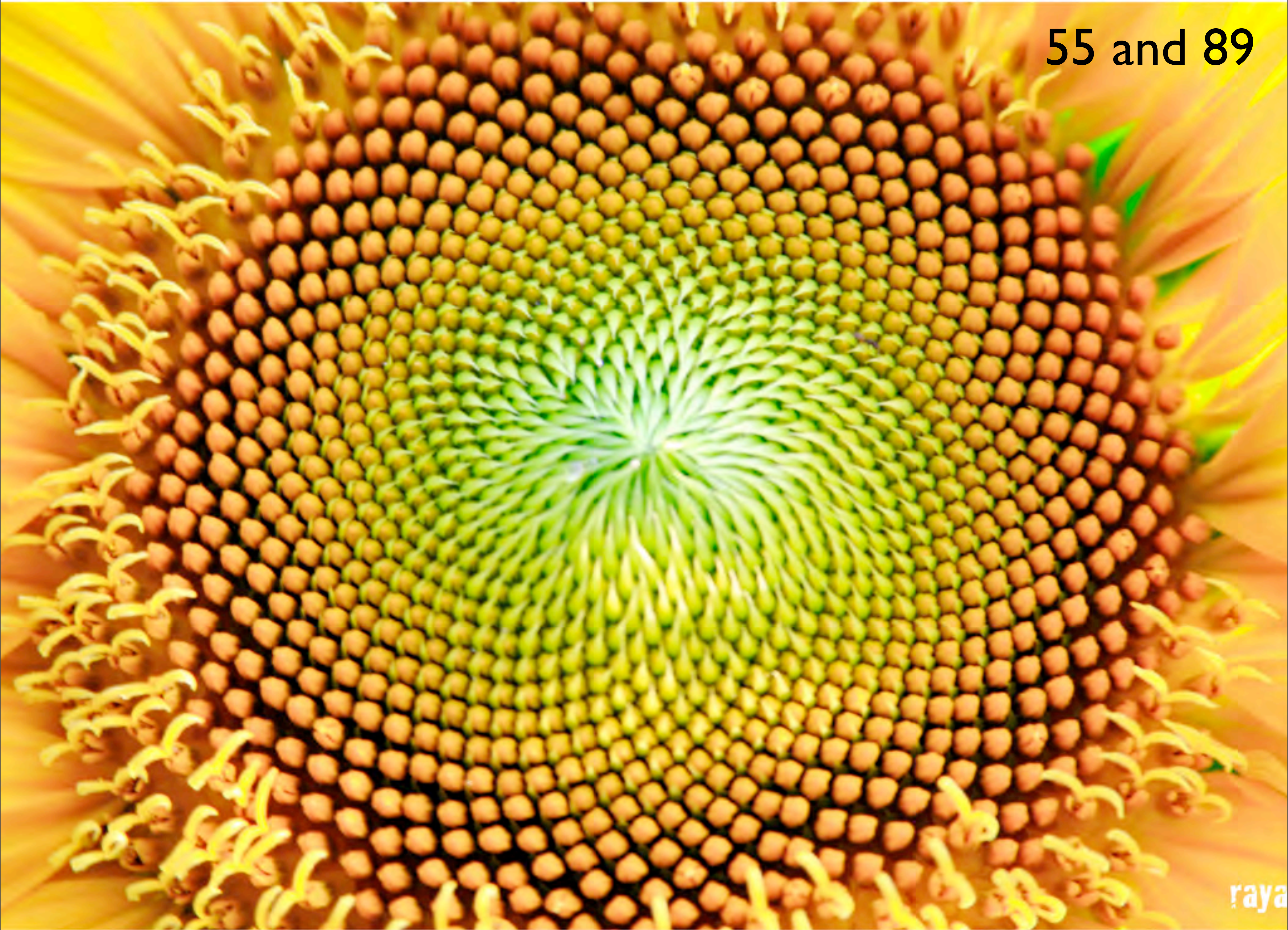
Pineapples

Artichokes

Pinecones

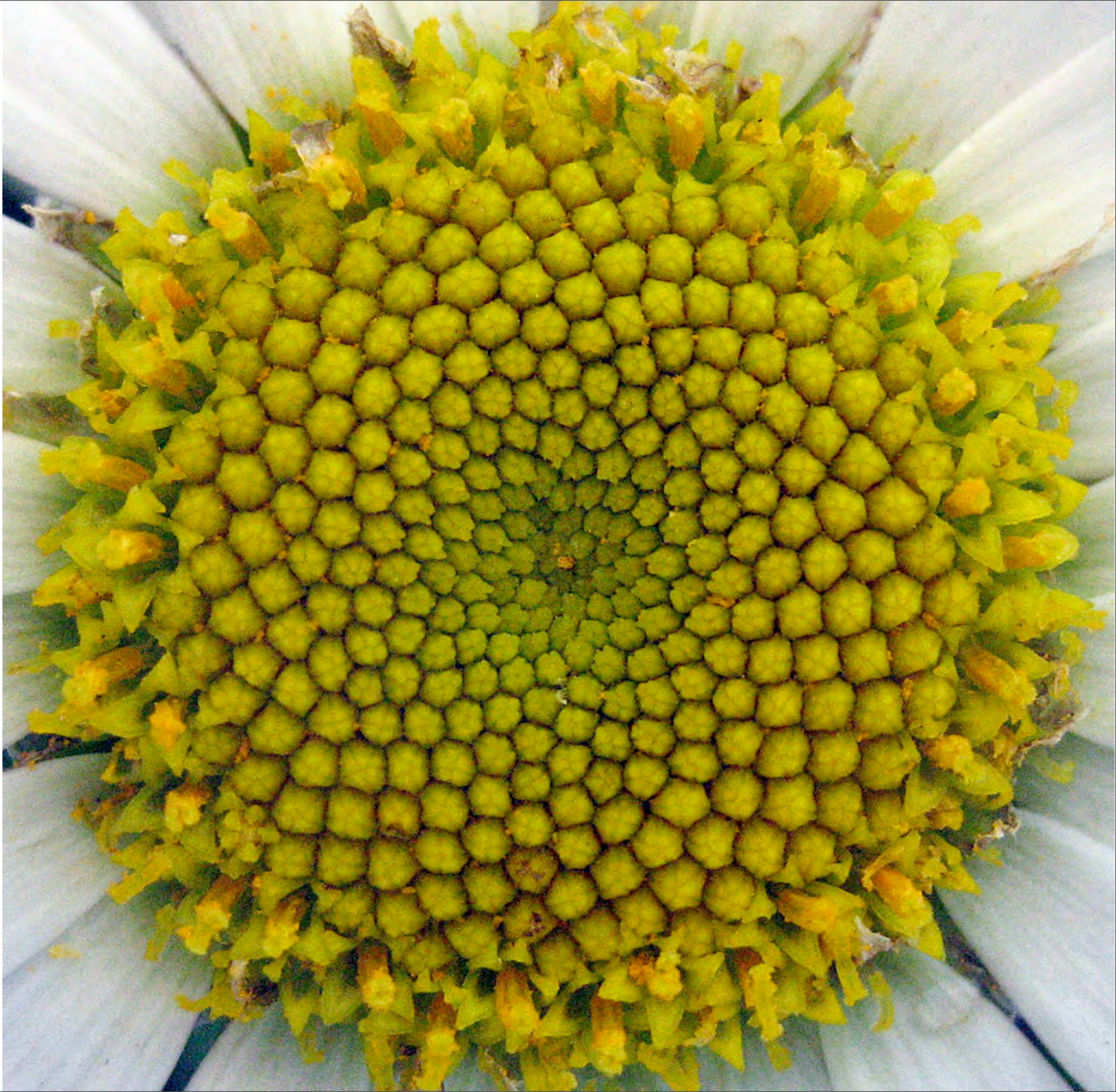
How many spirals...

21 and 34


55 and 89
raya

More Fibonacci Numbers

Yes,
Fibonacci
Numbers!



WHY?

Coincidence?
Because they're "Nature's special numbers"?
Nature is always efficient... what is efficient about using Fibonacci numbers?

A connection between Fibonacci numbers and The Golden Ratio

A ratio is a multiplicative relationship between two numbers.

Fractions are ratios.

$$\frac{22}{7} \qquad \frac{355}{113}$$

Ratios of Fibonacci Numbers

$\frac{144}{89} = 1.6179$

$\frac{89}{55} = 1.6181...$

$\frac{55}{34} = 1.6176...$

$\frac{34}{21} = 1.6190...$

$\frac{21}{13} = 1.6153...$

$\frac{13}{8} = 1.625$

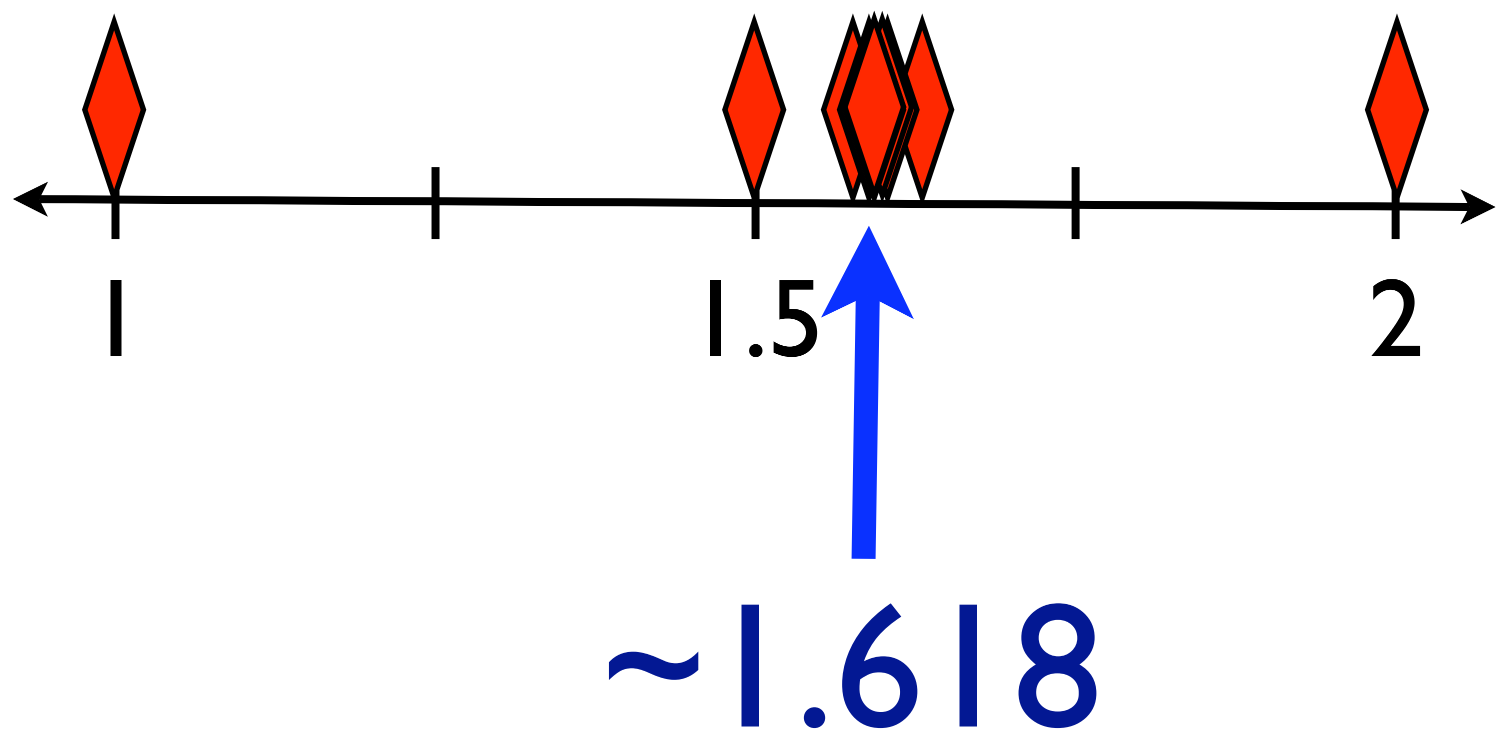
$\frac{8}{5} = 1.6$

$\frac{5}{3} = 1.6666...$

$\frac{3}{2} = 1.5$

$\frac{2}{1} = 2$

$\frac{1}{1} = 1$



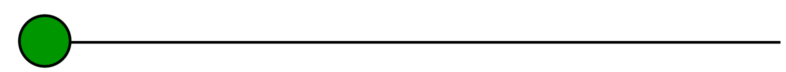
1.618033988749895....

The ratios of Fibonacci Numbers
converge on
THE GOLDEN RATIO.

Plant "use" the Golden Ratio to distribute their leaves, or petals, or branches, or seeds.

A spiral with angle = 1/8 of revolution

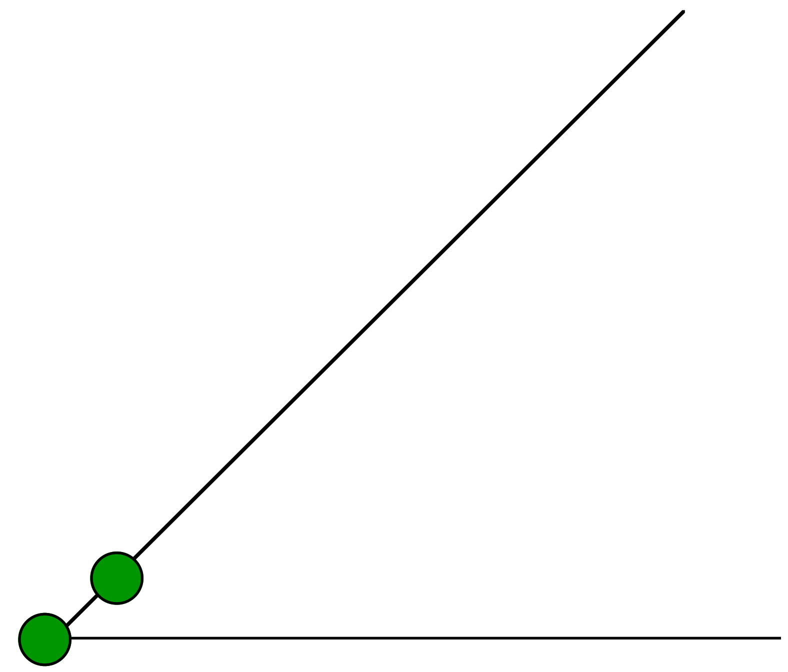
$1/8 \times 360^\circ = 45^\circ$



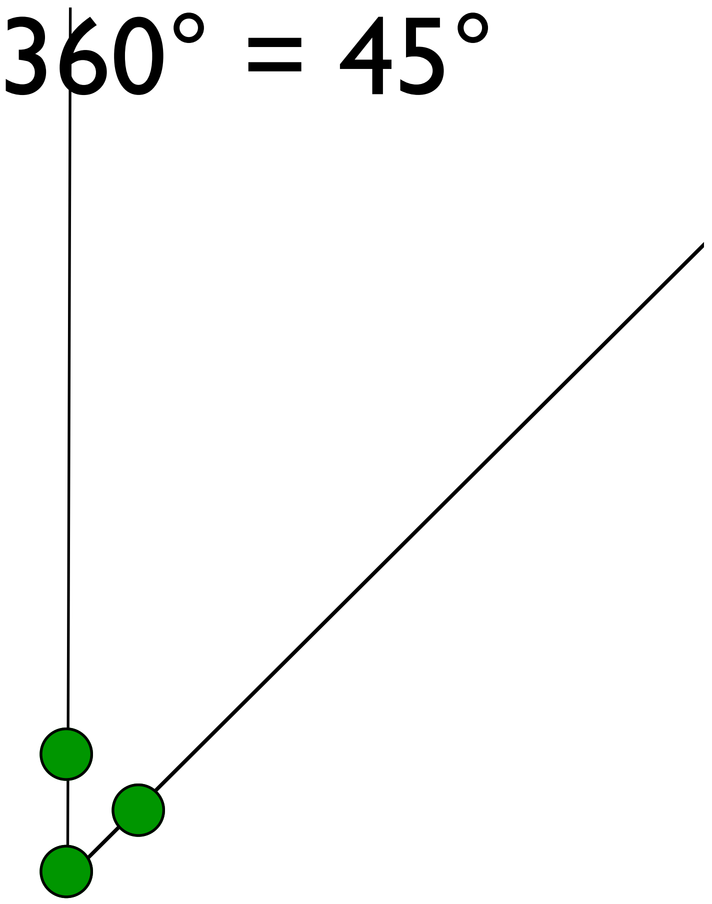
Many plants produce leaves or petals or seeds so that the angle between a new part and the previous part is always the same.

This makes the spirals we see.

A spiral with angle = 1/8 of revolution
$1/8 \times 360° = 45°$

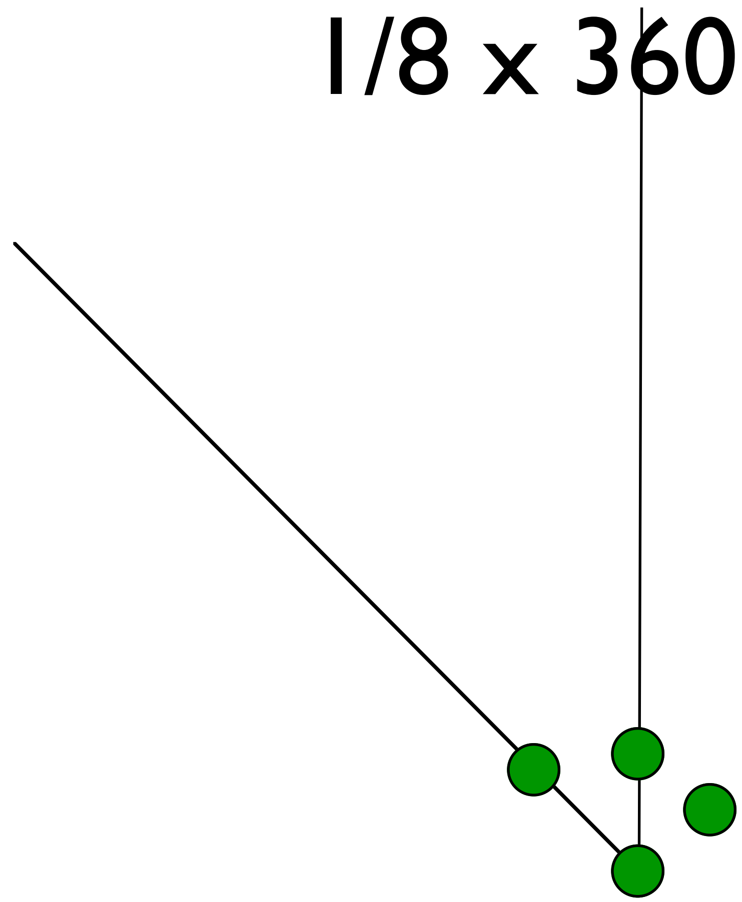


A spiral with angle = 1/8 of revolution
1/8 x 360° = 45°

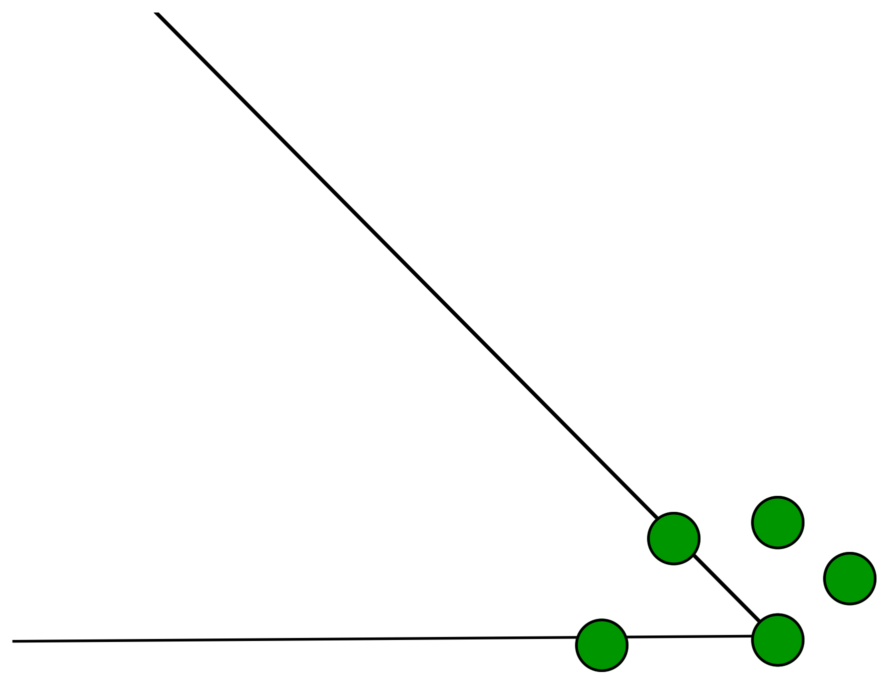


A spiral with angle = 1/8 of revolution

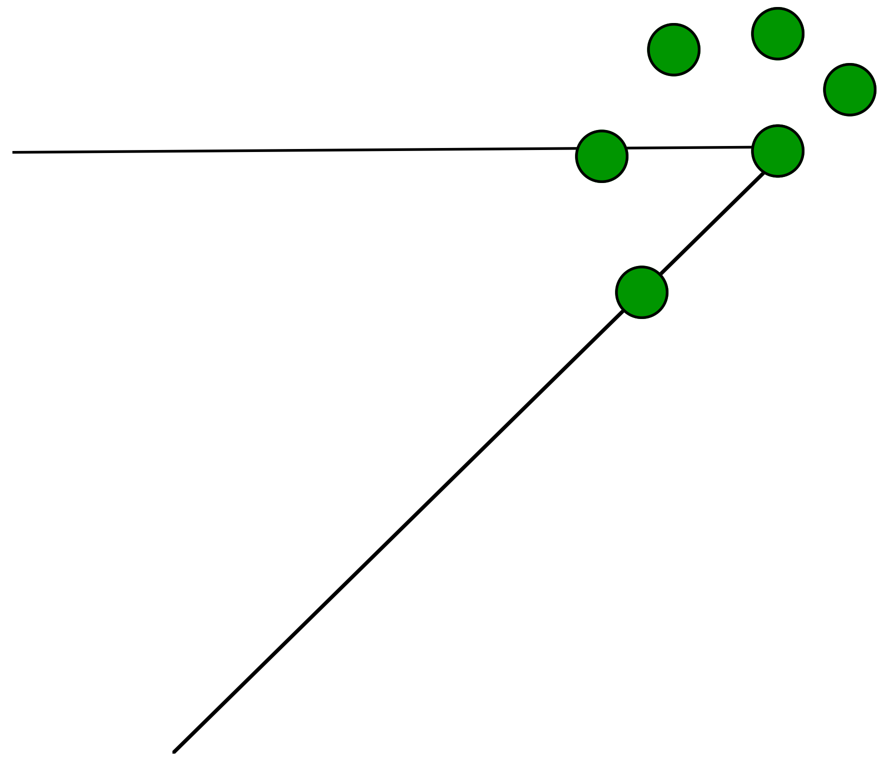
$1/8 \times 360° = 45°$



A spiral with angle = 1/8 of revolution
1/8 x 360° = 45°

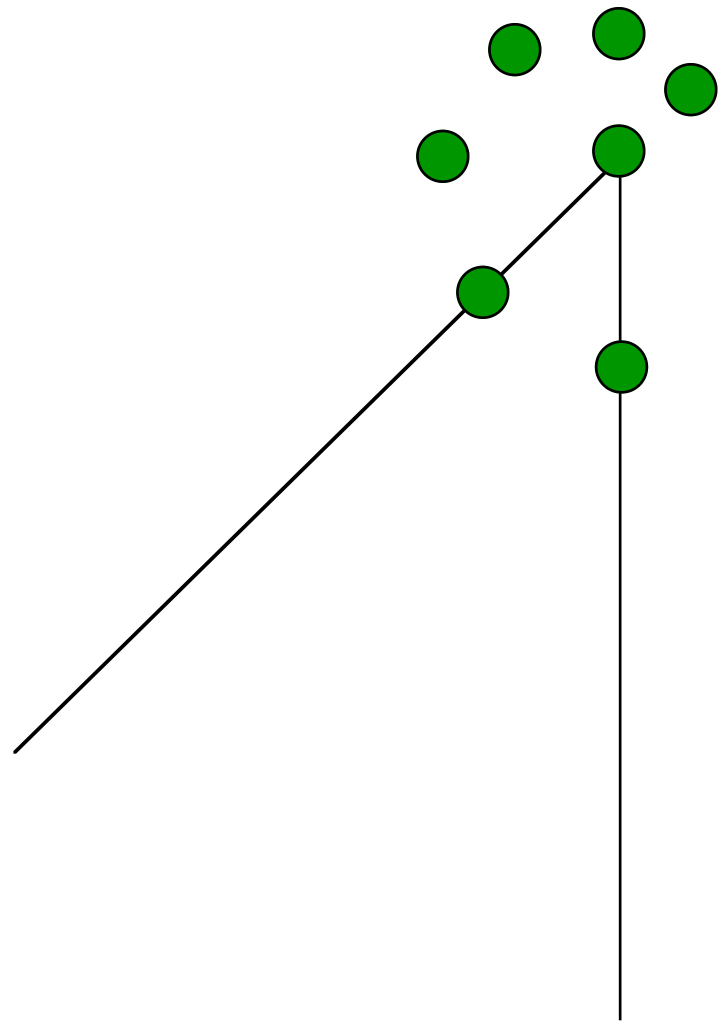


A spiral with angle = 1/8 of revolution
$1/8 \times 360° = 45°$



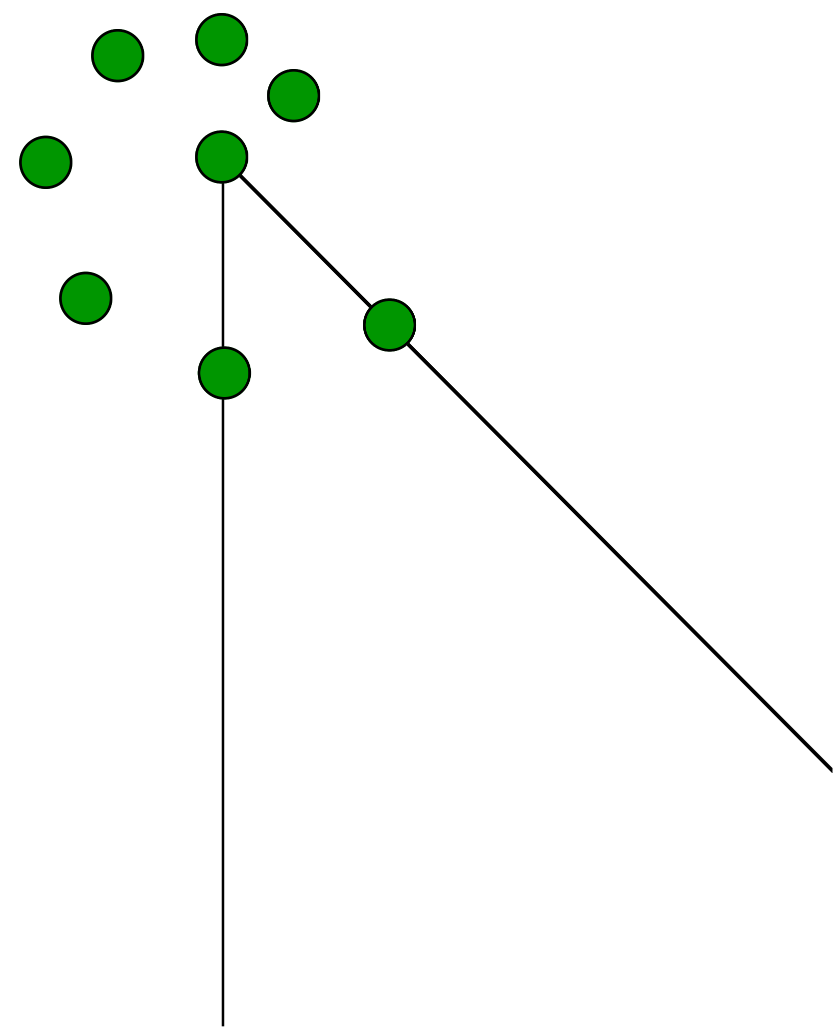
A spiral with angle = 1/8 of revolution

$1/8 \times 360° = 45°$

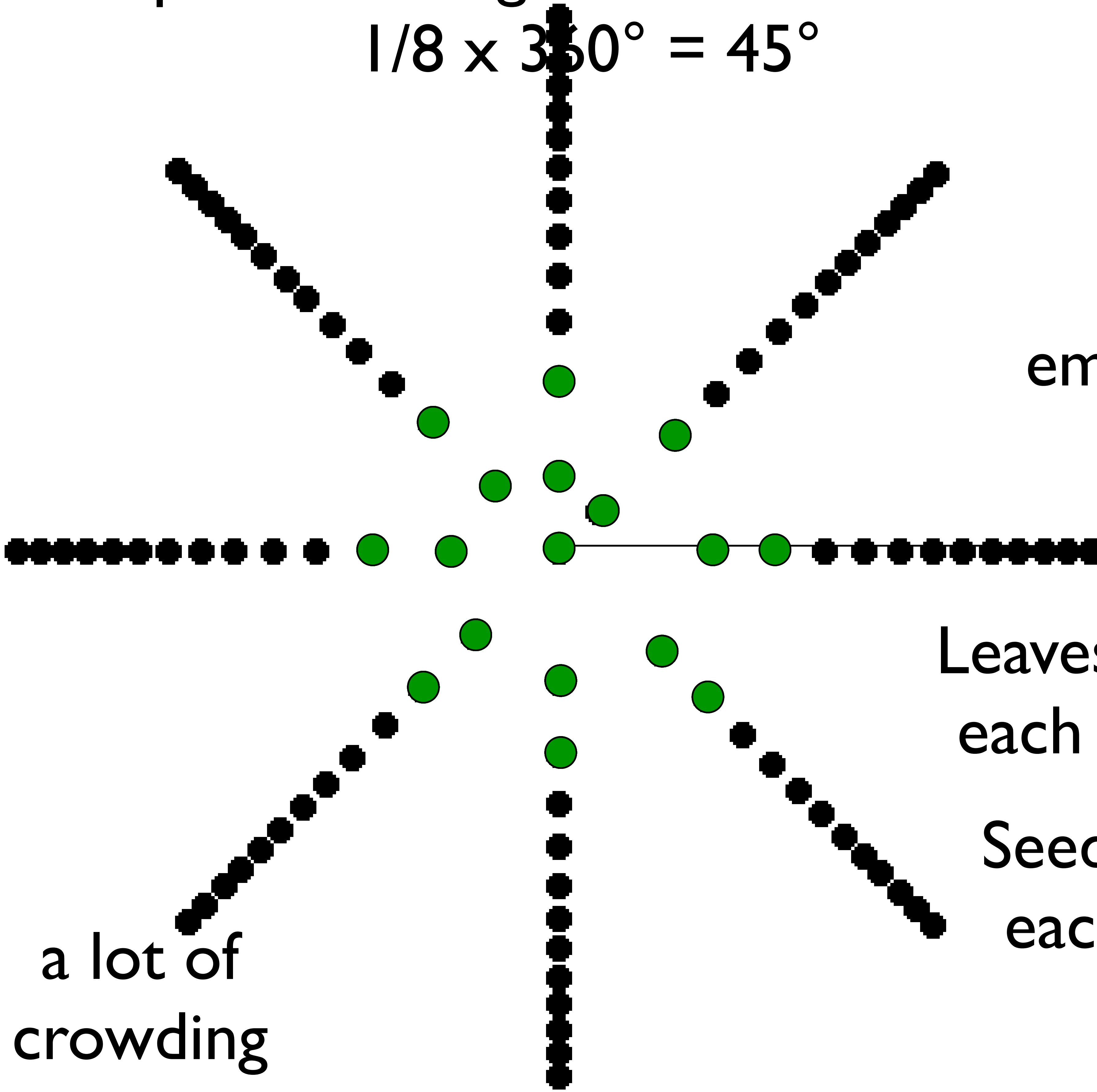


A spiral with angle = 1/8 of revolution

$1/8 \times 360° = 45°$



A spiral with angle = 1/8 of revolution
$1/8 \times 360° = 45°$



a lot of empty space

Leaves block each other.

Seeds crowd each other.

a lot of crowding

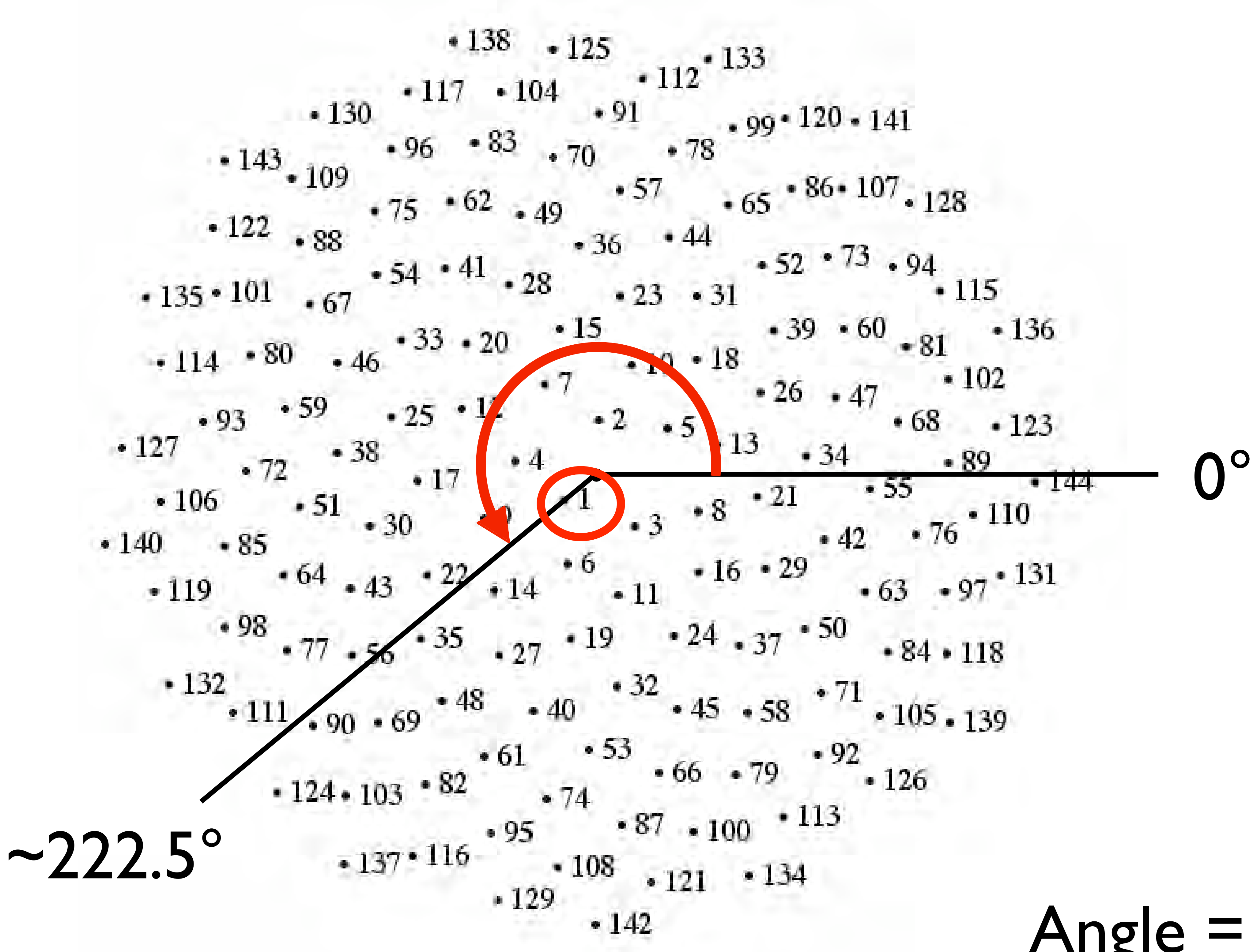
Instead of a 45° angle between seeds,
plants use Φ x 360°
= 582.5° ....

$$\begin{array}{r} 582.5^\circ \\ -\ 360^\circ \\ \hline =\ 222.5^\circ \end{array}$$

222.5° is called the Golden Angle.

It is 1.618 full turns.

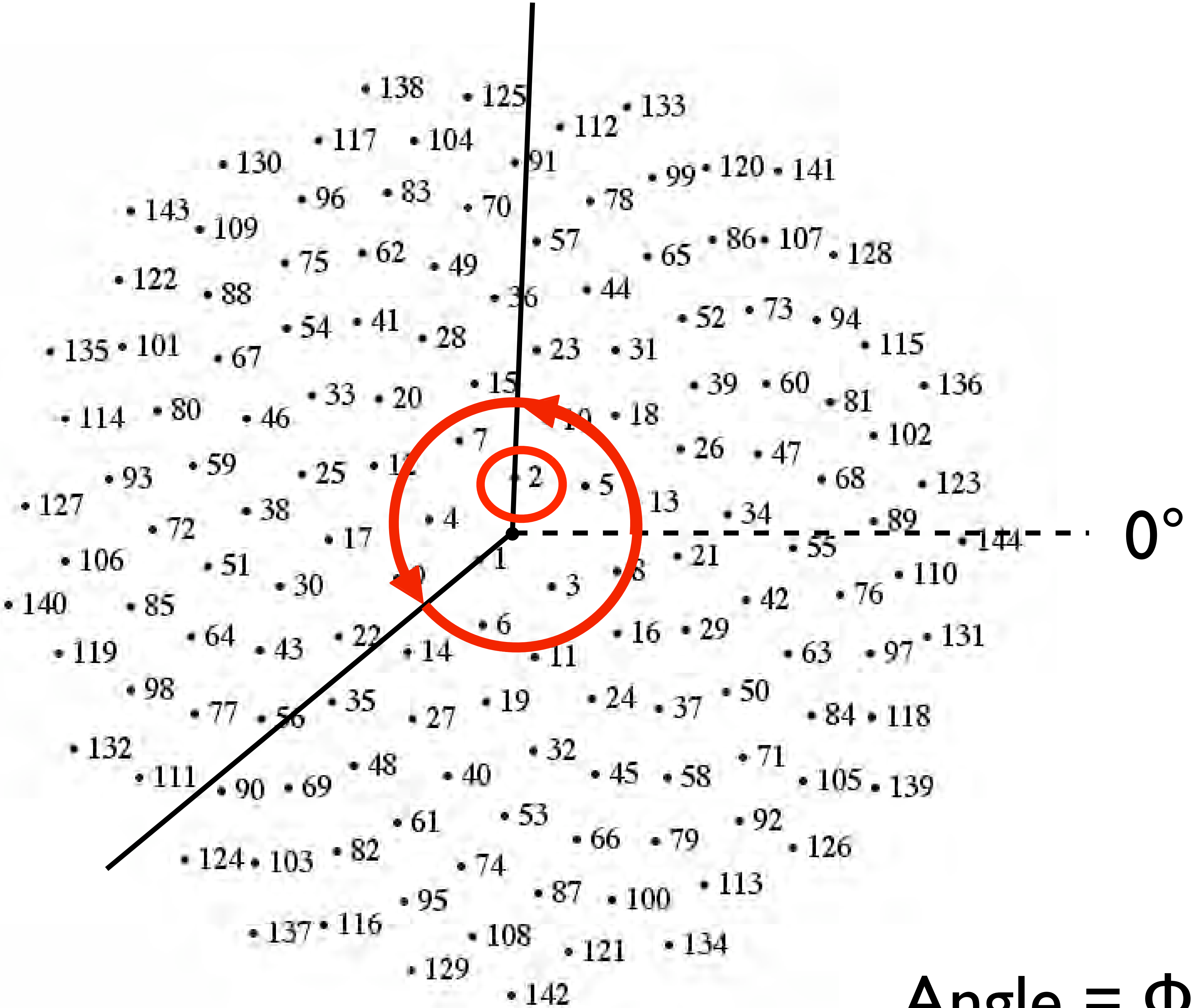
How seeds are arranged in nature



Seed 1

Angle = 222.5°

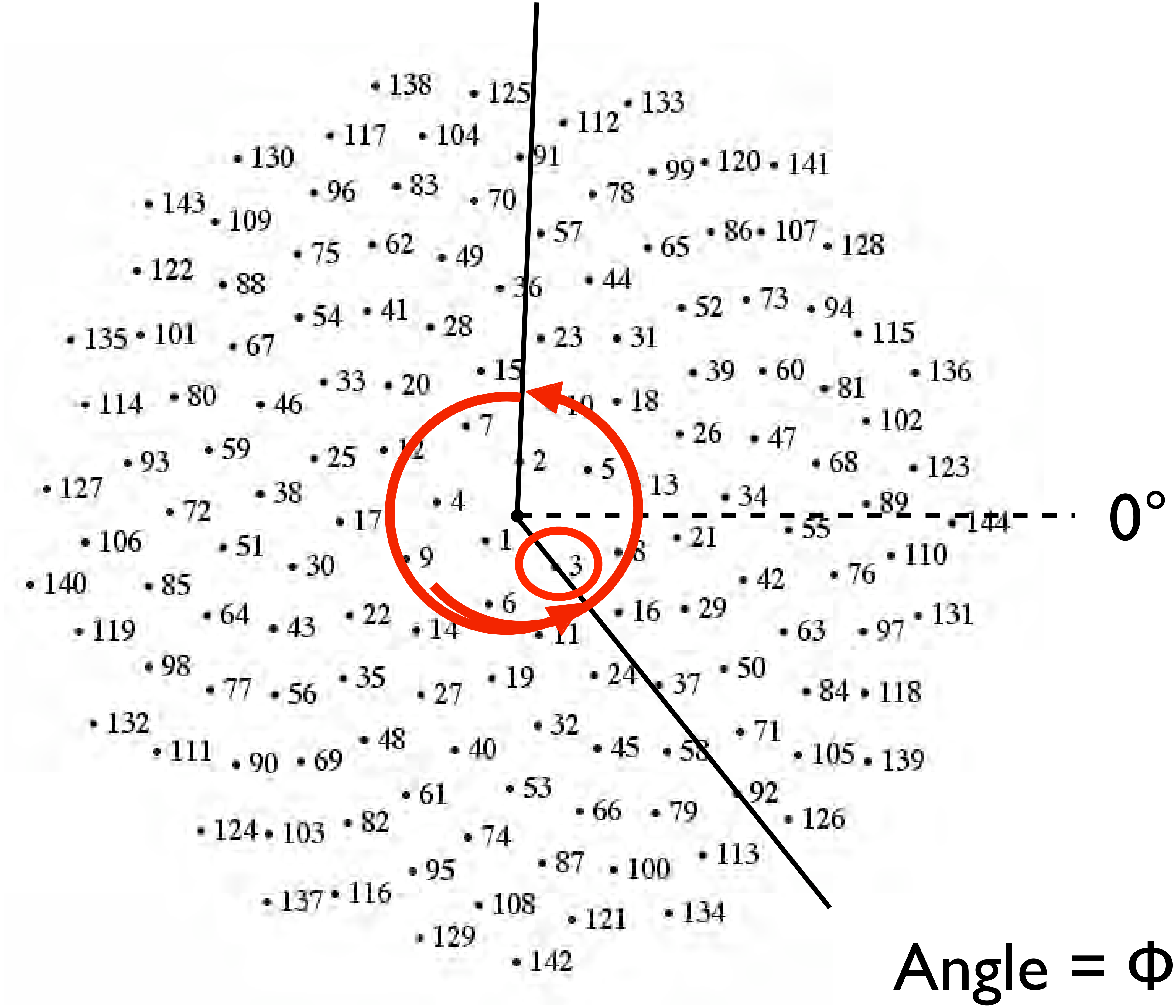
Angle = Φ revolutions



Seed 2

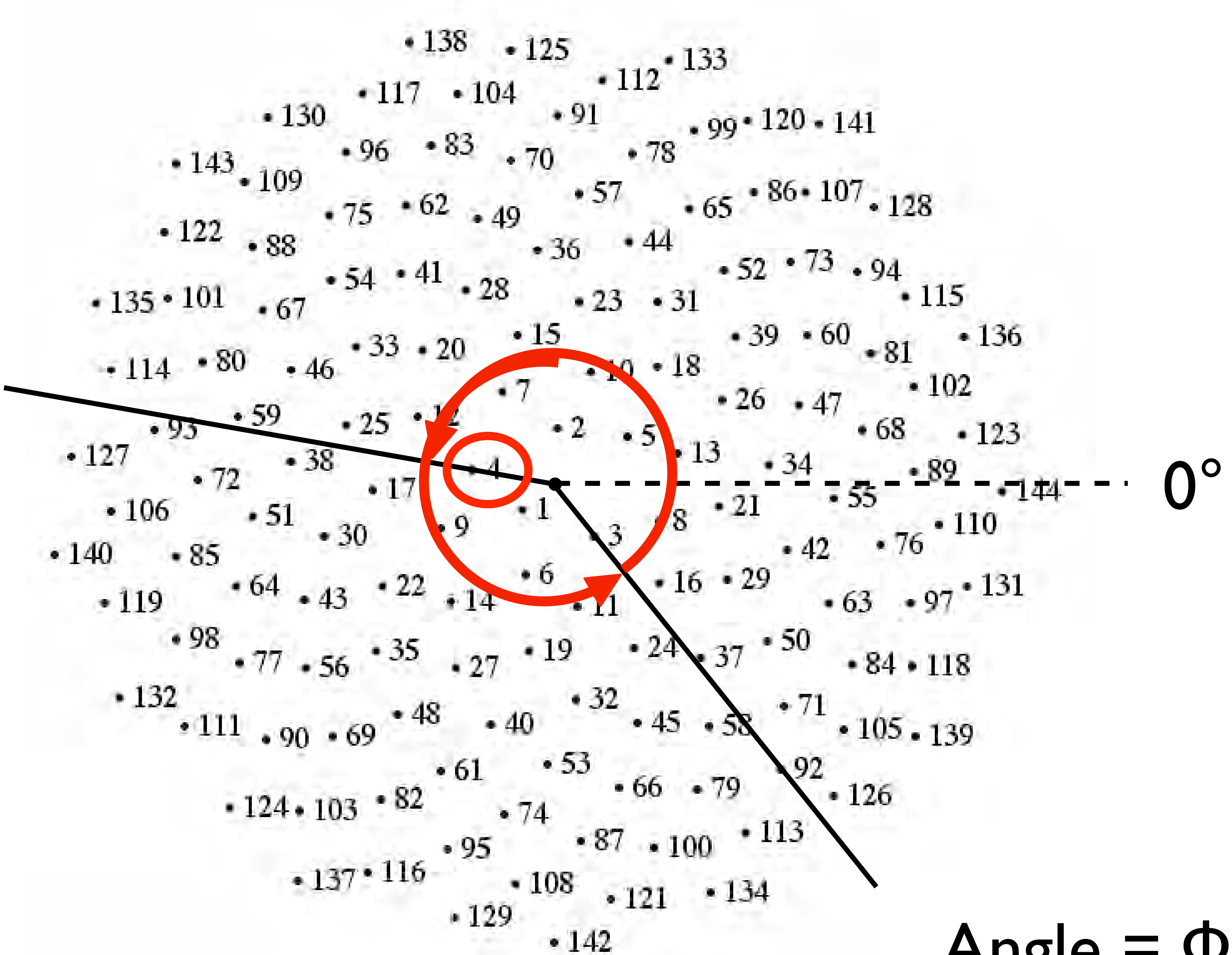
Angle = 222.5° x 2

Angle = Φ x 2 revolutions



Angle = Φ x 3 revolutions

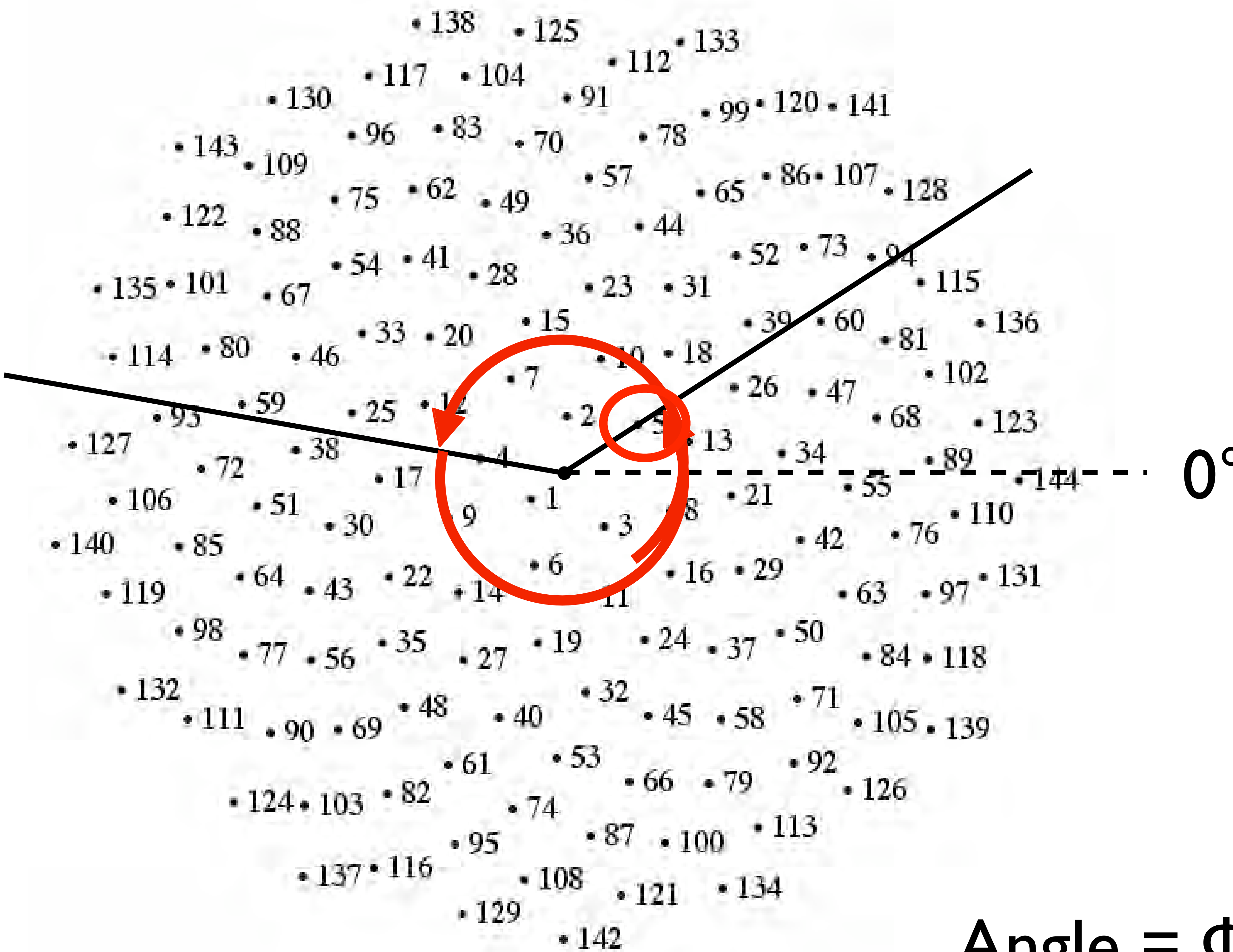
Seed 3 Angle = 222.5° x 3



Angle = Φ x 4 revolutions

Seed 4

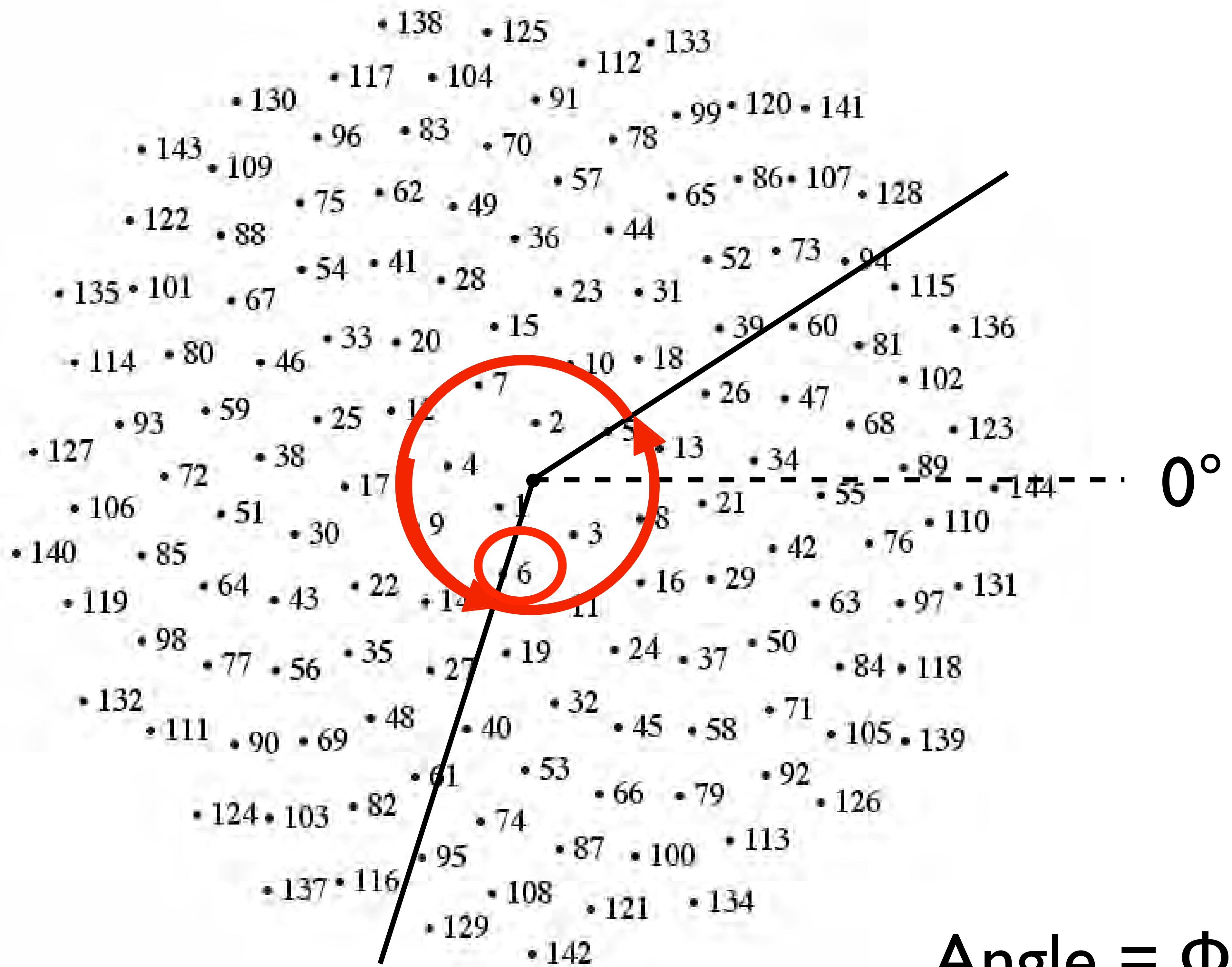
Angle = 222.5° x 4



Angle = $\Phi \times 5$ revolutions

Seed 5

Angle = 222.5° x 5

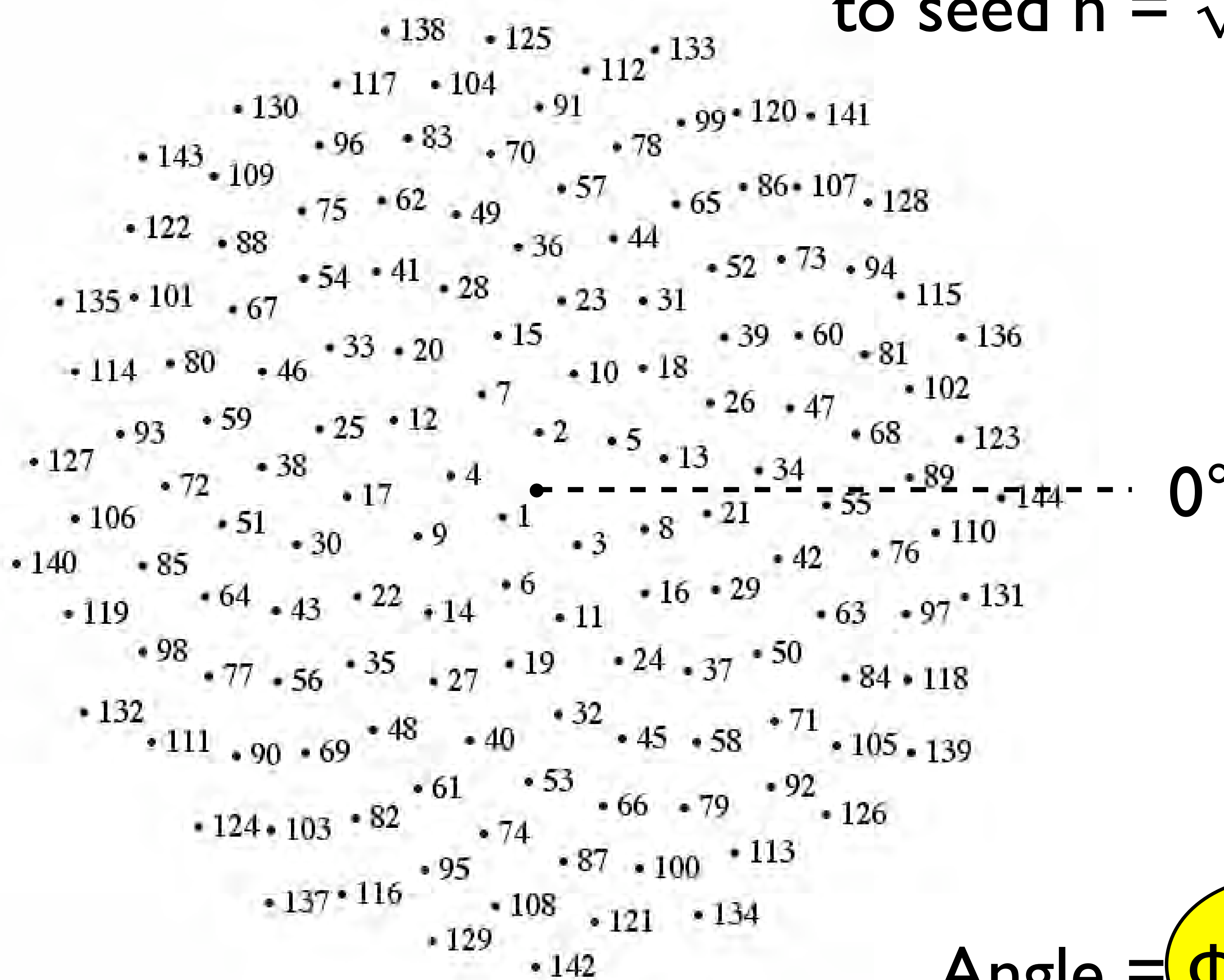


Angle = Φ x 6 revolutions

Seed 6

Angle = 222.5° x 6

(distance from center to seed n = $\sqrt{n}$)

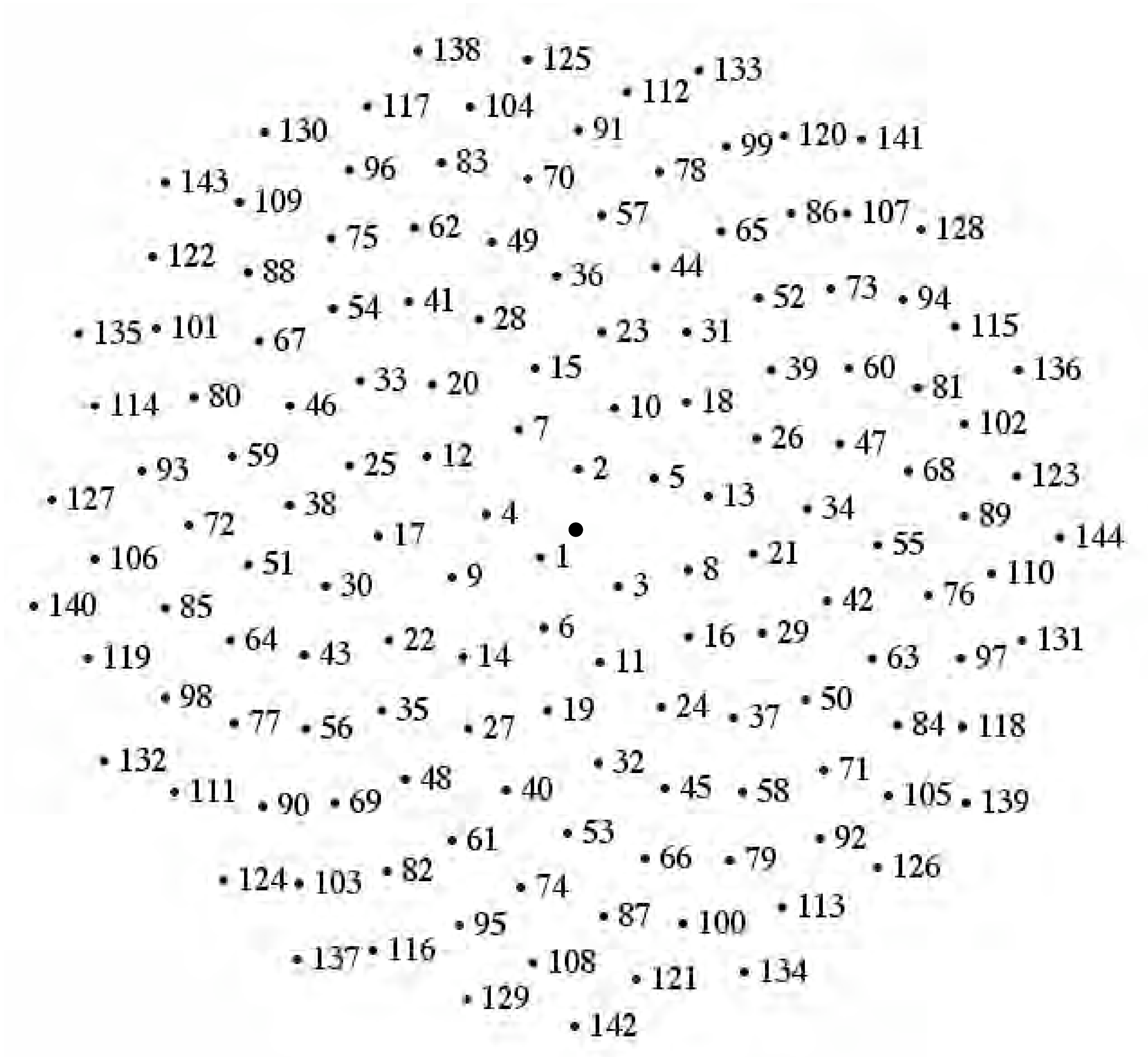


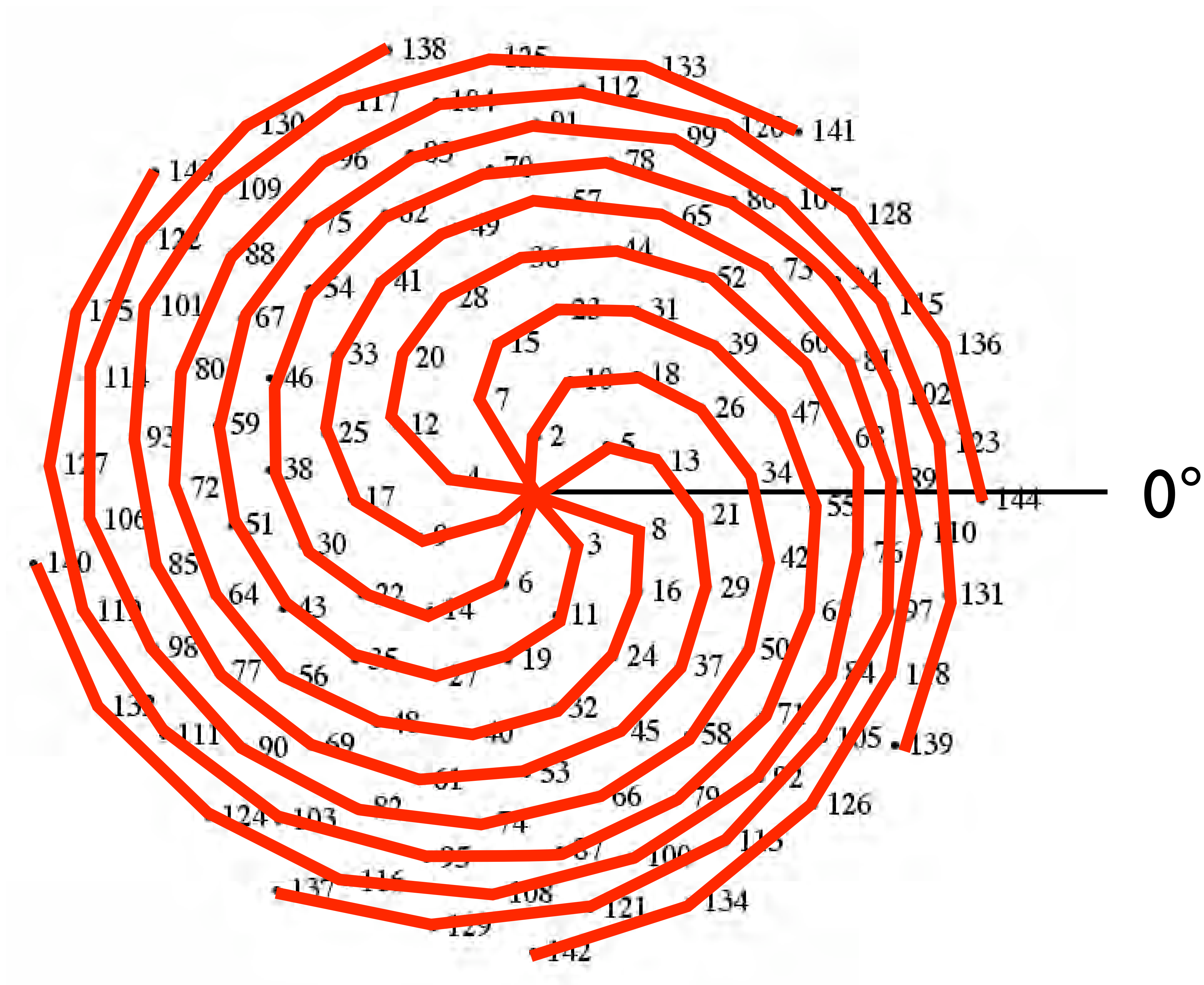
Angle = Φ x n revolutions

Seed n

Angle = 222.5° x n

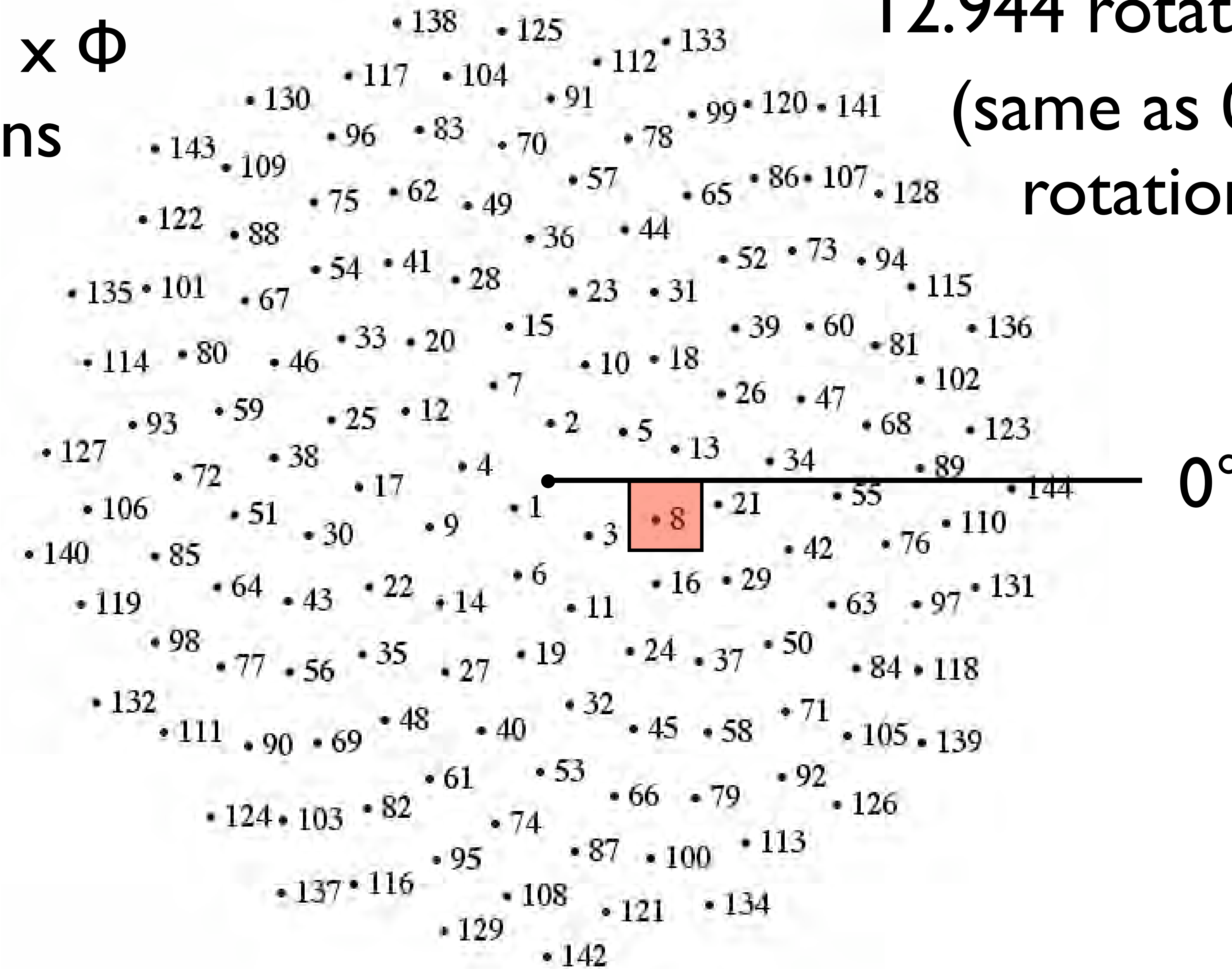
Find spirals, find patterns...




0°

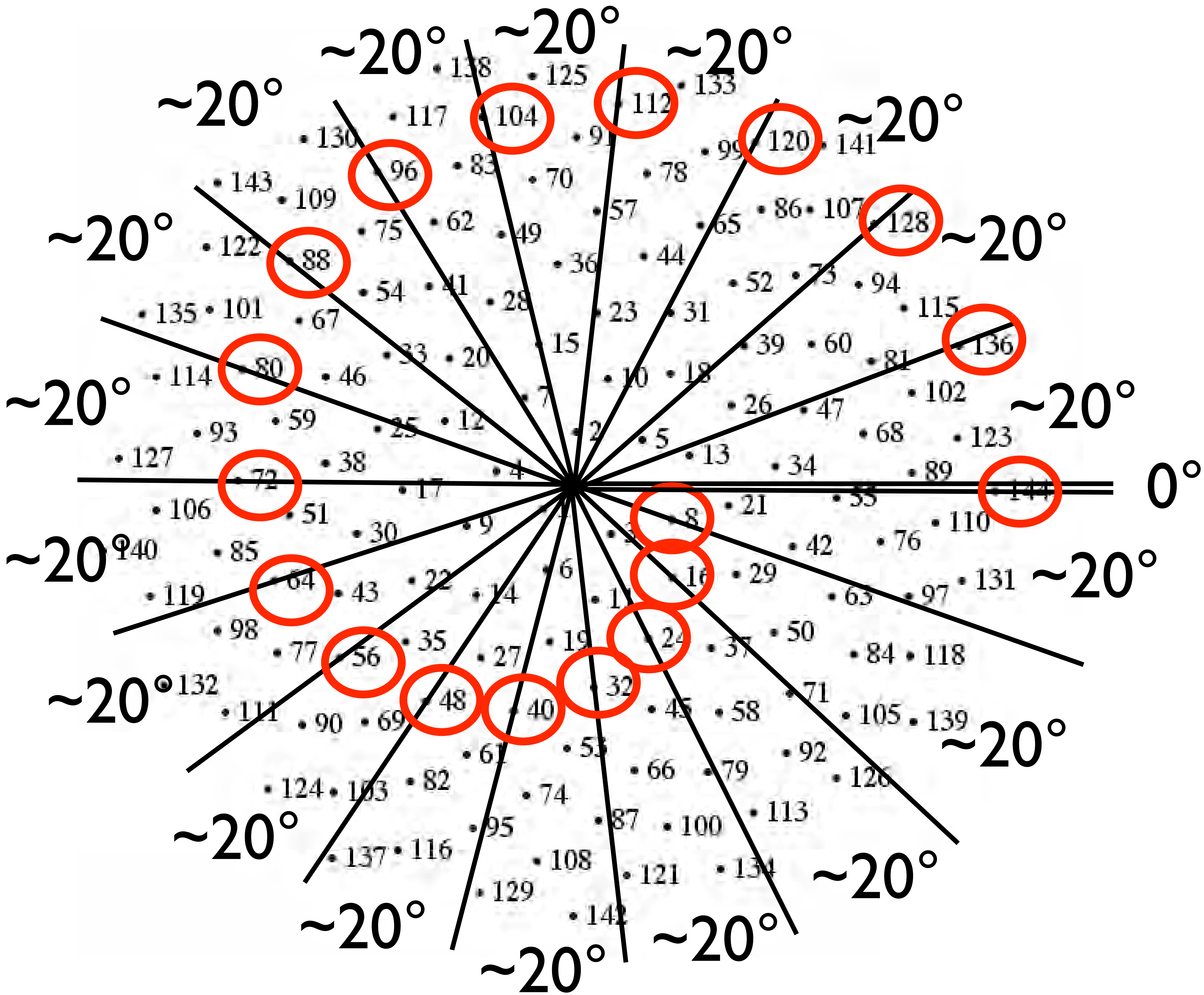
Seed 8:
angle = 8 x Φ rotations

12.944 rotations (same as 0.944 rotations)

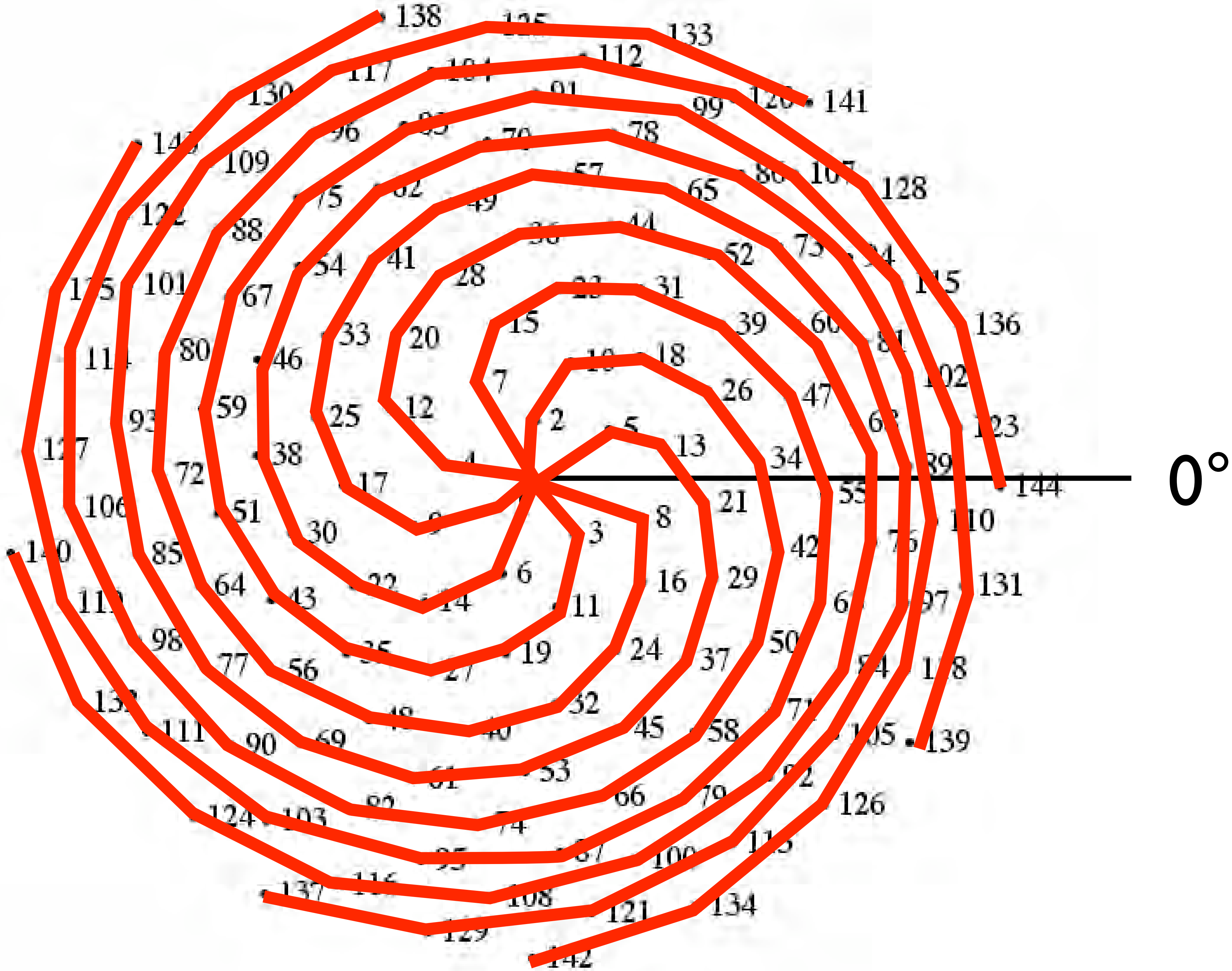


0.944 x 360° = 340°

... which is about 20° short of of 360°



~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
~20°
0°


0°

Every eighth seed: $8 \times \Phi$ rotations

$$\sim 8 \times \frac{13}{8} \text{ rotations} = \sim 13 \text{ rotations}$$

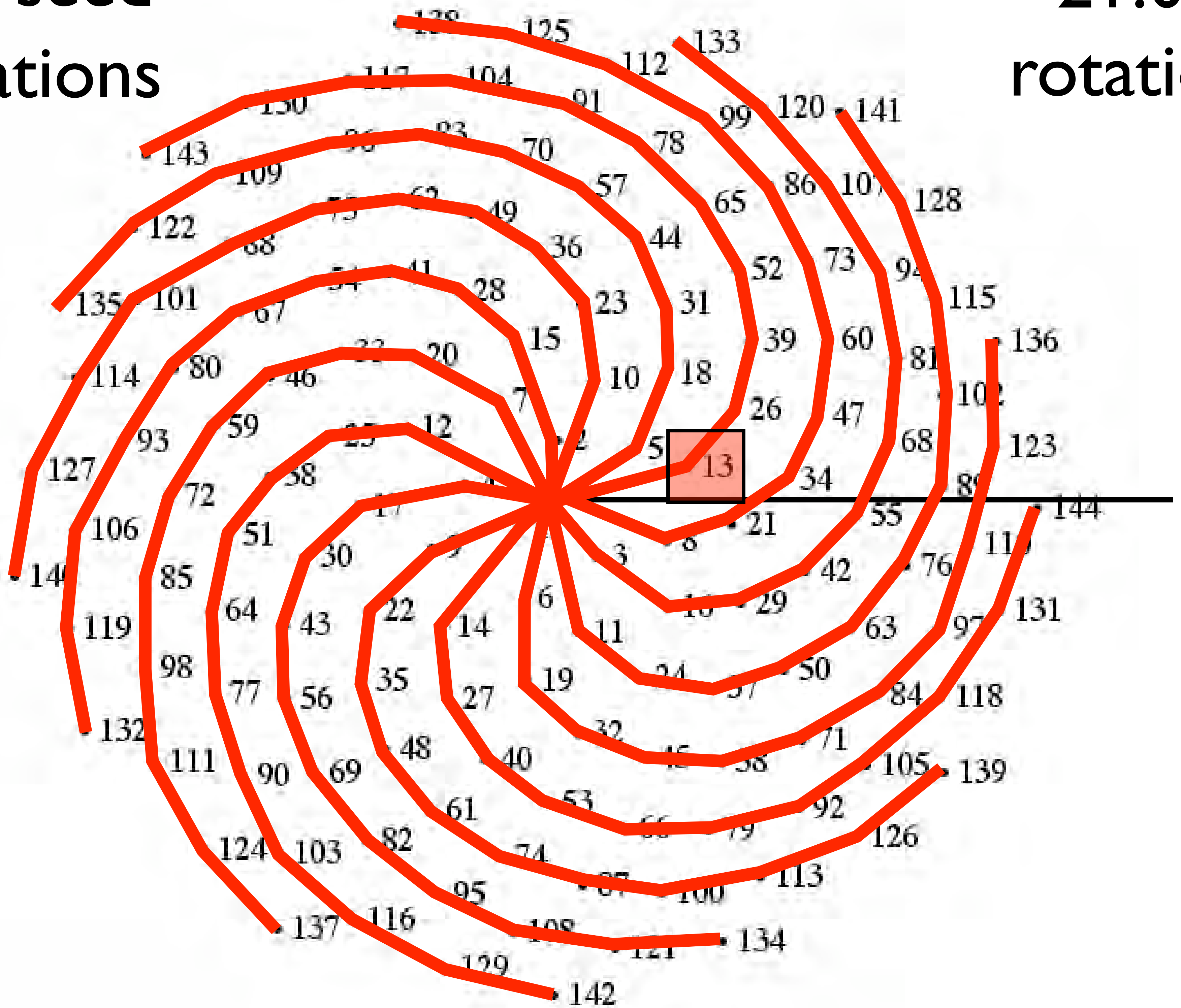
Nearly a whole number of rotations

Every thirteenth seed: $13 \times \Phi$ rotations

$$\sim 13 \times \frac{21}{13} \text{ rotations} = \sim 21 \text{ rotations}$$

Every 13th seed
13 x Φ rotations

= 21.034...
rotations



= 12.4° past the previous seed in its spiral

138 125 133 112
130 117 104 91 99 120 141
143 109 96 83 70 78
75 62 49 57 65 86 107 128
122 88 36 44
135 101 67 54 41 28 23 31 52 73 94 115
114 80 46 33 20 15 10 18 39 60 81 136
7 26 47 102
127 93 59 25 12 2 5 13 68 123
38 4 34 89
72 17 1 8 21 55 144
106 51 30 9 3 110
140 85 42 76
119 64 43 22 14 6 11 16 29 63 97 131
98 77 56 35 27 19 24 37 50 84 118
132 111 90 69 48 40 32 45 58 71 105 139
61 53 66 79 92 126
124 103 82 74 87 100 113
95 137 116 108 121 134
129 142
0°


Number of Points
100...
Show Labels
Show 0-Line
Angle
0.618033989
OK
Rotations...
Preset Angles
- select -
Spiral Families
No spirals...
13
OK
Send feedback to
Nathan Shields
10minutemath.com
Golden Spirals
This program is based on the the arrangement of seeds in a sunflower. Points are plotted in an outward spiral, with a constant angle between consecutive points. The radius is proportional to the square root of the point number.
The sunflower uses the Golden Ratio (one of the preset angles) to achieve the most efficient arrangement of seeds. Other angles tend to produce noticable "arms" where the points are clumped together.
When more than 400 points are plotted, the "show all spirals" and "show labels" options are disabled.
OK

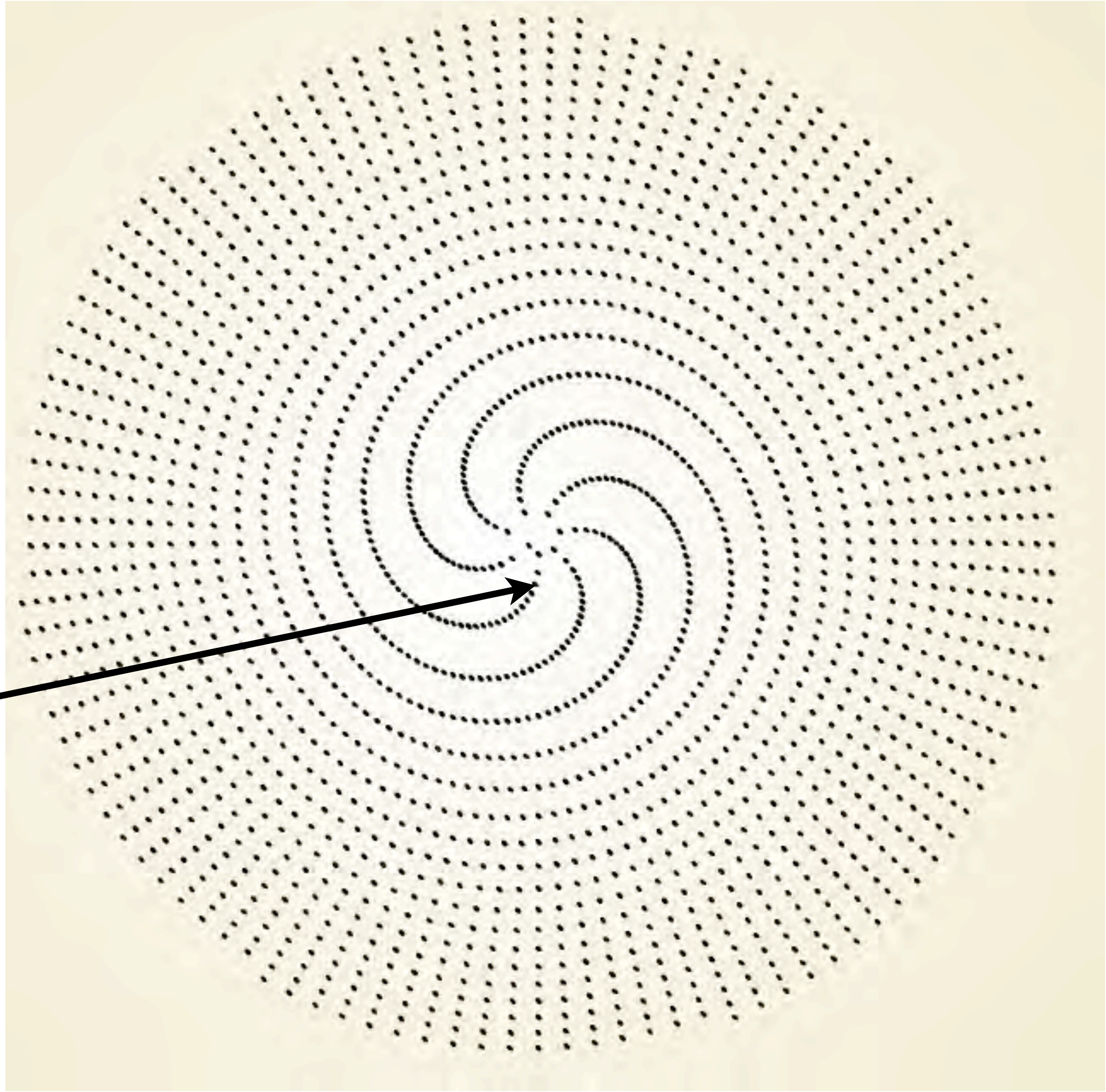
Why is Φ and not some other number like π?

Seed 7 is rotated 7 x π revolutions = 21.9911 turns
= about 357° = 3° from start...VERY CLOSE!

$\pi = 3.14159265358979...$

$\pi \cong \frac{22}{7}$

(3.142857....)

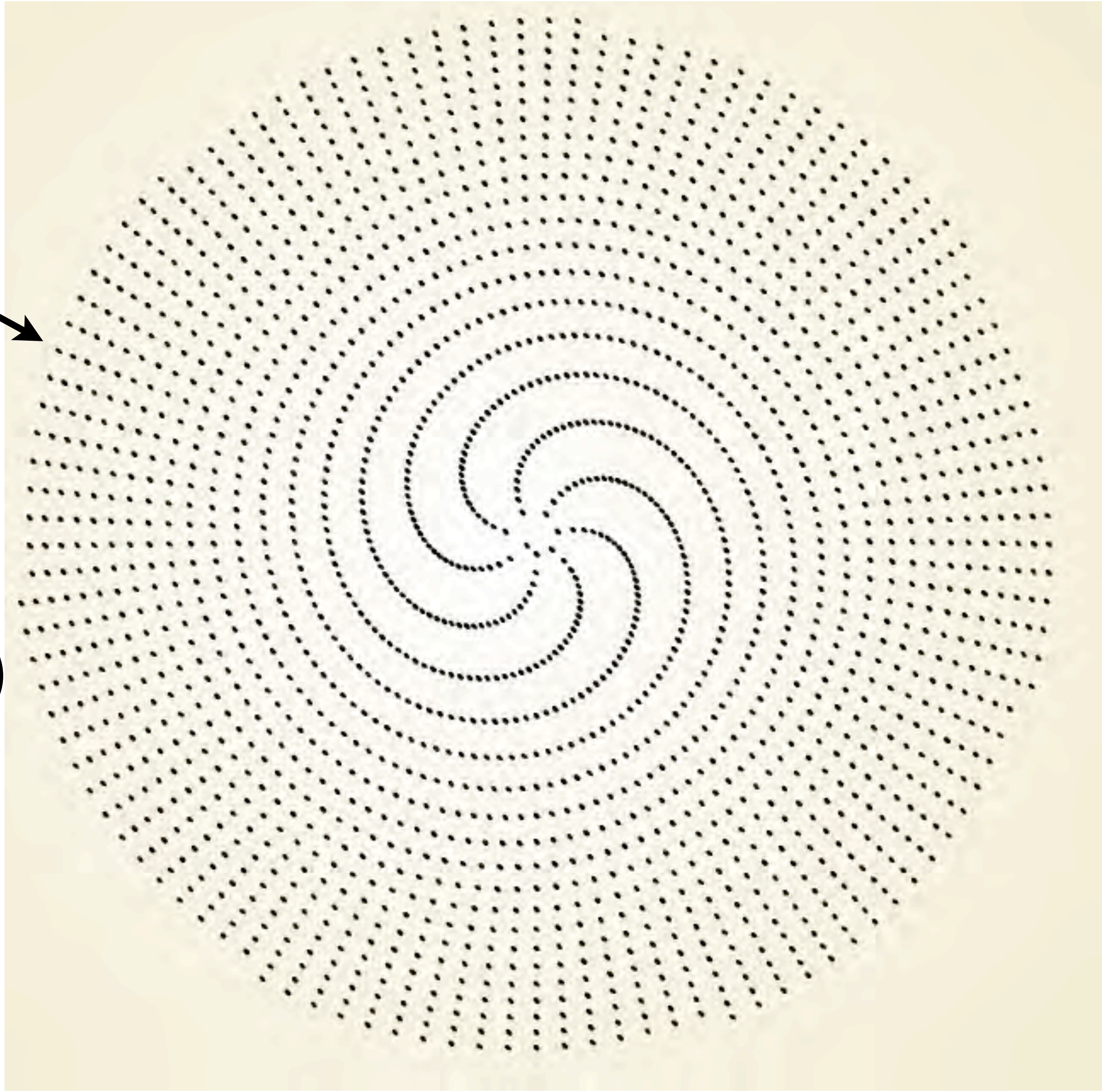


7 arms

Compare: Seed 8 in the Φ spiral is 20° off...
much worse for making spirals.

Seed 113 is rotated 113 x π revolutions = 354.9999699 turns = about 359.989° = 0.011° from start!

113 arms



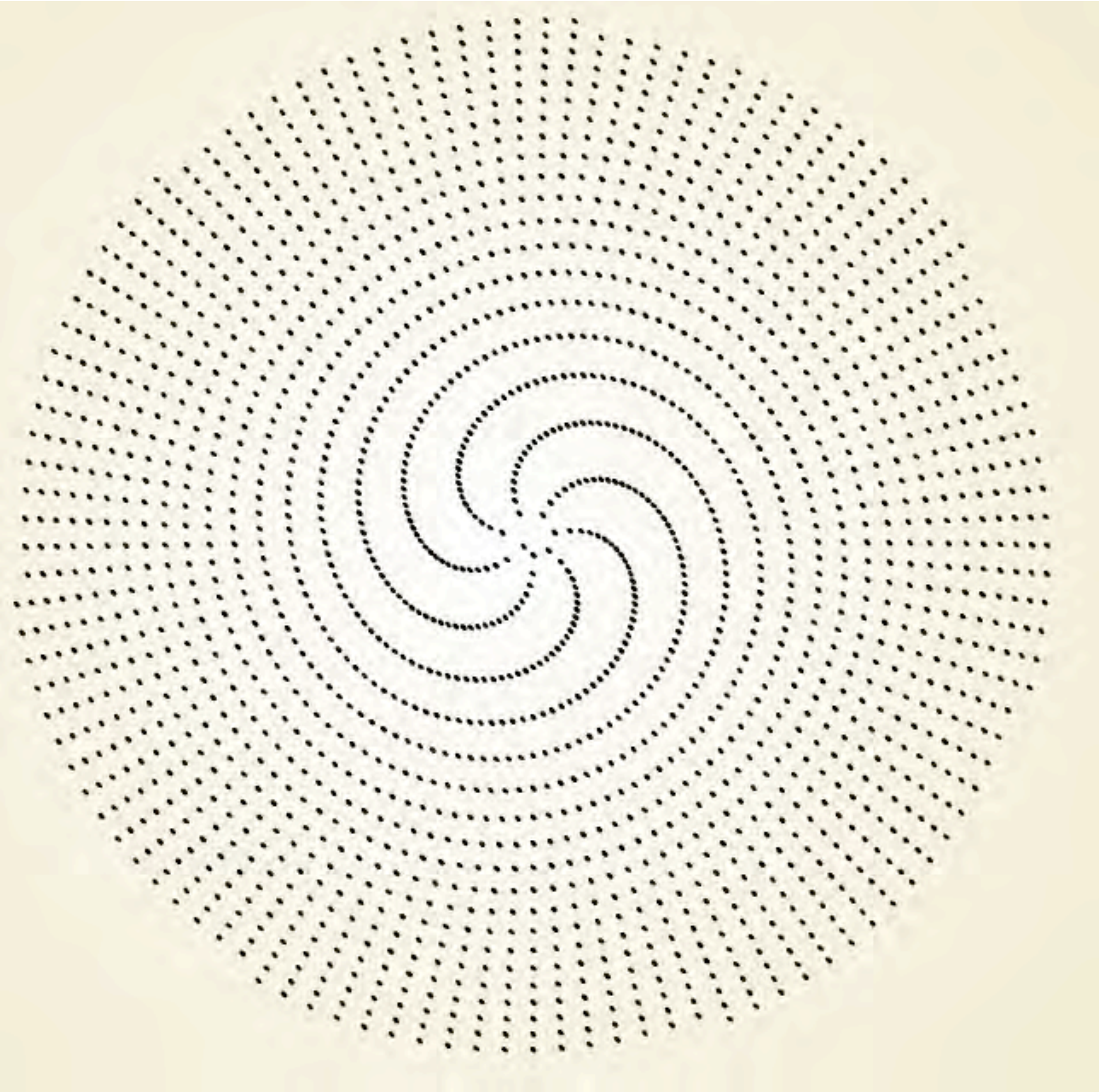
$$\pi \cong \frac{355}{113}$$

(3.14159292...)

Pi has GOOD approximations.
Good approximations make strong spiral arms.
Strong spiral arms cause crowding and empty space.

$\frac{22}{7}$

Good approximation!

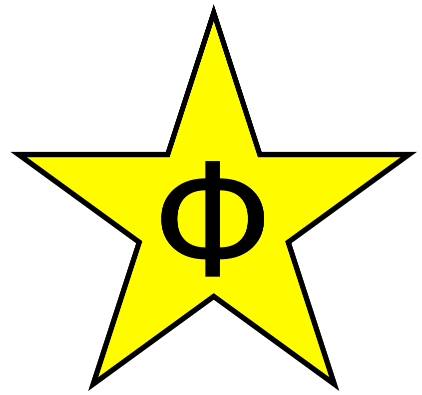


$\frac{355}{113}$

GREAT approximation!

Good rational approximations = Bad seed distributions

The number with the worst possible approximations is...



The Golden Ratio is the **Most Irrational** Number...
The Golden Ratio gives the best seed placement!

Note: This can be analyzed with Hurwitz's Theorem which gives a measure of how good or bad approximations are, and using **continued fractions** to generate convergents.

Every real number, rational or irrational, can be written as a continued fraction as follows:

$$N = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e + \dots}}}}$$

where a, b, c, d, e, ... are positive integers.

Here's the beginning of π as a continued fraction.

$$\pi \cong 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \ldots}}}}$$

The greater the integers, the better the rational approximations.

For fun...

$$\pi \cong \frac{355}{113} \quad (3.14159292035\ldots)$$

$$\pi \cong 3 + \frac{16}{113} = 3 + \frac{1}{\frac{113}{16}}$$

$$\pi \cong 3 + \frac{1}{7 + \frac{1}{16}}$$

$$\pi \cong 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1}}}$$

$$\Phi = 1 + \frac{1}{\Phi}$$

$$\Phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\Phi + \dots}}}}}$$

These are the lowest we can go...

Φ has the worst approximations!

How does this work in plants?

Mathematical Hypothesis:

Some plants twist uniformly as they grow.

The twist produces equal angles between leaves/branches/seeds.

The plant grows to a point which:

- maximizes light on leaves or
- minimizes stress (crowding) on seeds.

BIG IDEA

The resulting angle is *automatically* the Golden Ratio, and the Golden Ratio produces Fibonacci Numbers.

The mechanism may be

- active... feedback influences growth
- evolutionary... shaped over many generations
- or both.

The mechanisms differ in different plants, but the mathematical result is the same.

1. *The centric representation* 13

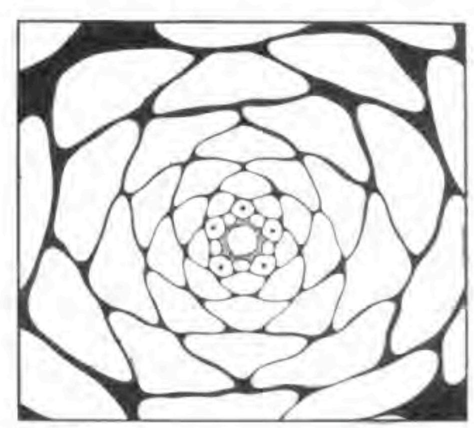
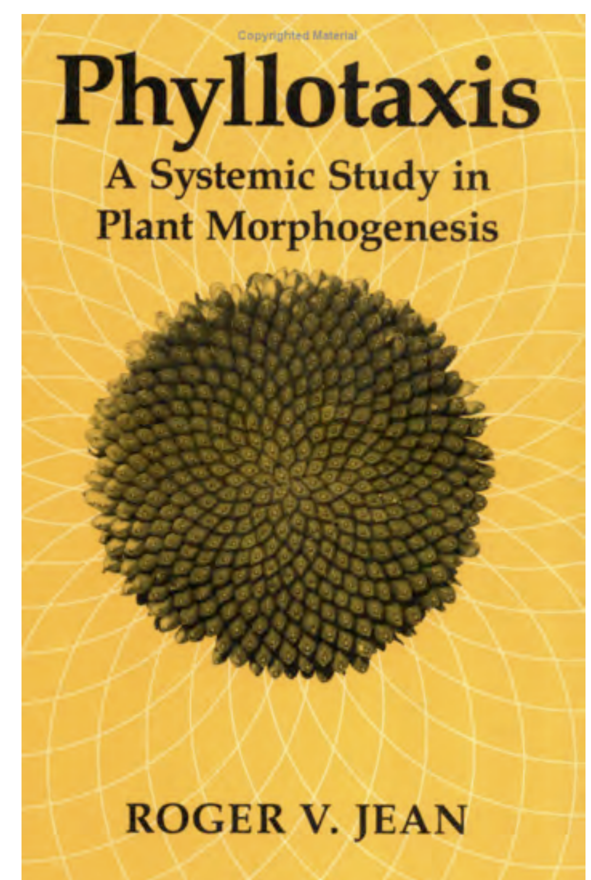


Figure 1.1. An alternating whorled system observed in a transverse section of a shoot of *Lycopodium selago* (fir club moss) showing five leaf primordia in a whorl. The dots in the middle of five primordia serve to identify the second whorl. The primordia arise at the center of the shoot, and they move outward by growth, while increasing in size. The system is denoted by (5, 5). There are *Lycopodium selago* with (5, 6), (4, 5), (4, 4) and (3, 3) parastichy pairs. (From Church, 1904b.)



CRAZY IDEA:

Nature seeks the most chaos.
From this chaos, comes amazing order.

FIN