Deciphering Nature’s Code

The Secret Mathematics of the Natural World

– by Mike Naylor –
Welcome!
The natural world is alive with beautiful and amazing shapes.
These forms are all connected by a single number. Mathematics’ most mysterious number.
Ancient people held this number in awe and reverence, and gave it names like “The Divine Proportion” and “The Golden Ratio.”
It has inspired some of the greatest art and architecture of all time.
Inspired some of the greatest art and architecture of all time
It is considered to be “the most beautiful number”
It’s found in many places in nature.
What is this number? Why is it so amazing?

Why does it show up often in nature?

Recent research has answered this question and revealed a deep connection between mathematics and science.
Euclid (c. 300 BCE)

Known as the “Father of Geometry”, he wrote a math book called *Elements* that is still used today — 2300 years later!
The Divine Proportion

Cut a segment into “mean” and “extreme” ratios.

\[
\frac{\text{longer part}}{\text{shorter part}} = \frac{\text{whole thing}}{\text{longer part}}
\]
\[
\frac{x}{1} \cdot x = \frac{x + 1}{x} \cdot x
\]

\[x^2 = x + 1\]

If you square this number, it's the same as adding 1.

\[
\begin{align*}
1 & \Rightarrow 1^2 \overset{?}{=} 1 + 1 \\
2 & \Rightarrow 2^2 \overset{?}{=} 2 + 1 \\
3 & \Rightarrow 3^2 \overset{?}{=} 3 + 1 \\
1.5 & \Rightarrow 1.5^2 \overset{?}{=} 1.5 + 1 \\
2.25 & \neq 2.5
\end{align*}
\]
Hurray for the Quadratic Equation!

\[ x^2 = x + 1 \]
\[ x^2 - x - 1 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ a = 1 \quad b = -1 \quad c = -1 \]

\[ x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2} \]
\[ x^2 = x + 1 \]

\[
\frac{1 \pm \sqrt{5}}{2} = 1.618033988749895\ldots
\]

Sooooooooooooooooooooooooooooo beautiful!

\[ 1.618033988749895^2 = 2.618033988749895\]
\[
\frac{x^2}{x} = \frac{x + 1}{x} \times \frac{x}{x}
\]

\[
x = 1 + \frac{1}{-1 - 1}
\]

\[
x - 1 = \frac{1}{x}
\]

\[
\frac{1}{1.618033988749895} = 0.618033988749895
\]
This number is so important, it gets its own letter:

the Greek letter \textit{phi},
written like this: \( \Phi \)
How can you use a number like 1.618... in art?

And why would you want to?

In ancient times, people thought of numbers as shapes ...

... and $\Phi$ makes some of the most amazing shapes.
Φ does number tricks:

\[
\frac{1}{\Phi} = 0.618033988749895 = \Phi - 1
\]

\[
\Phi = 1.618033988749895
\]

\[
\Phi^2 = 2.618033988749895 = \Phi + 1
\]

Φ does geometry tricks too....
Golden Rectangle

φ (~1.618)

Square

smaller Golden Rectangle
Golden Rectangle

This is golden, too!
Golden Rectangle

\[ \phi \]

\[ \frac{1}{\phi} \]

\[ 0.618 \]
Golden Rectangle

Ratio of long \( \frac{\text{long}}{\text{short}} \) = \( \frac{\phi}{1} \) = \( 1 \) = \( \frac{1}{\phi} \)

This small rectangle is also golden!

short side \( \times \Phi = \) long side
\[ r = e^{-kt} \]
The ratio of short/long:

\[
\frac{x-1}{1} = \frac{1}{x}
\]

Solving for \(x\):

\[
x - 1 = \frac{1}{x}
\]

\[
x = \Phi
\]
Golden Triangle

36° 72° 36°

Φ

72° 36° 72°

Golden!
Phidias (ca. 500 BCE), the greatest sculptor of Classical Greece, used this ratio extensively.
It is reported than in many of Phidias’ statues:

\[
\frac{\text{Height of Figure}}{\text{Height of Belly Button}} = \Phi = 1.618...
\]

It was thought that this was the most beautiful position for one’s belly button.
It is also said that his work contained many golden rectangles.

\[
\frac{\text{length}}{\text{width}} = \Phi
\]

Statue of Zeus, one of the 7 wonders of the ancient world, built by Phidias.
Phidias’ work once filled the Parthenon, which seems to contain many golden rectangles.
Φ is named for Phideas!

Φιδίας
Throughout the ages, many artists have used the golden ratio, most famously Italian Renaissance man Leonardo DaVinci (ca. 1500)
Many famous artists have intentionally used the golden ratio in their art.

DaVinci  

Michaelangelo  

Raphael  

Dali  

Seurat  

Mondrian
But... are these shapes really more beautiful?

Many psychological studies have been done over the past 200 years...

and there is no conclusive evidence that people prefer this ratio.
There are many claims that the ratio can be found all over the body...
... and that the most beautiful faces conform closely to the golden ratio.

(as if we don’t have enough reasons to feel bad about how we look!)
Is it true?

Is this ratio part of the way our bodies are assembled?
Distance A
Distance B
= 1.6
Golden Ratio!
WOW!
Distance A
Distance B

= 1.6

Golden Ratio again!

WOW!
Distance A
Distance B

= 2

(never mind, ignore that...)
Wait a minute...
22 points

# lengths?
# ratios
(≠1)?
22 points

\[22 \times 21 \div 2 = 231 \text{ lengths}\]

\[231 \times 230 = 53,130 \text{ ratios}\]
I looked at 50,000 ratios between 0 and 20, and some of them were close to 1.6! Amazing?

Not very amazing.
There are many other claims about the golden ratio that are
  • coincidences
  • wishful thinking
  • bad approximations
There are many other claims that are

- coincidences
- wishful thinking
- bad approximations
If you’re looking for it, you can find it!
You can also find any other number you want.
Be wary! It is not enough to find a few measurements that are “close” to the ratio.

**Ask: Is there a reason?**

The Golden Ratio DOES appear in some places in nature.

The reasons are amazing.
The reasons are MATHEMATICAL.
Leonardo Pisano
(ca. 1170 – 1250)

Leonardo of Pisa

Son of Bonacci

Fibonacci

Leaning Tower of Pisa
(ca. 1174 – present)
Fibonacci’s Travels
(ca. 1180–1200)
Liber Abaci
“The Book of Calculating”
Also in the book: “The Rabbit Problem”

The solution method produces numbers that are extremely important in nature.
January

February

March

April

May

June

December?
<table>
<thead>
<tr>
<th>Pairs of:</th>
<th>Babies</th>
<th>Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Feb</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Mar</td>
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<tr>
<td>Apr</td>
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</tr>
<tr>
<td>Nov</td>
<td>34</td>
<td>55</td>
</tr>
<tr>
<td>Dec</td>
<td><strong>55</strong></td>
<td><strong>89</strong></td>
</tr>
</tbody>
</table>

**Total**: 144
The sequence is called \textit{the Fibonacci sequence}.

It was so named by Edouard Lucas. (1842–1871, Paris)
Bee Families

Fertilized Egg = Female

Unfertilized Egg = Male
Bee Family Tree

= M

= F

Draw the next two steps!
Sneezewort
Many plants often have Fibonacci numbers in them.
21 petals each
34 petals on this sunflower

34 petals on this Gerbera flower
Trilliums have 3 petals, Buttercups and pansies have 5 petals, Delphiniums have 8 petals, Marigolds & Black-eyes Susans & Ragwort have 13 petals, or 21 petals or 34 petals.
Plant Spirals

Artichokes

Pineapples

Pinecones
How many spirals...

21 and 34
Count the spirals... 55 and 89
More Fibonacci Numbers
Yes,
Fibonacci
Numbers!
WHY?

Coincidence?
Because they’re “Nature’s special numbers”?
Nature is always efficient... what is efficient about using Fibonacci numbers?
A connection between Fibonacci numbers and The Golden Ratio
A ratio is a multiplicative relationship between two numbers.

Fractions are ratios.

\[
\frac{22}{7} \quad \frac{355}{113}
\]
Ratios of Fibonacci Numbers

\[
\begin{align*}
144 &= 1.6179 \\
89 &= 1.6181... \\
55 &= 1.6176... \\
34 &= 1.6190... \\
21 &= 1.6153... \\
13 &= 1.625 \\
8 &= 1.6 \\
5 &= 1.6666... \\
3 &= 1.5 \\
2 &= 2 \\
1 &= 1
\end{align*}
\]

\[\sim 1.618\]
1.618033988749895....

The ratios of Fibonacci Numbers converge on THE GOLDEN RATIO.

Plant “use” the Golden Ratio to distribute their leaves, or petals, or branches, or seeds.
A spiral with angle $= \frac{1}{8}$ of revolution

$\frac{1}{8} \times 360^\circ = 45^\circ$

Many plants produce leaves or petals or seeds so that the angle between a new part and the previous part is always the same.

This makes the spirals we see.
A spiral with angle $= \frac{1}{8}$ of revolution
$\frac{1}{8} \times 360^\circ = 45^\circ$
A spiral with angle $= \frac{1}{8}$ of revolution

$\frac{1}{8} \times 360^\circ = 45^\circ$
A spiral with angle = 1/8 of revolution

\[
\frac{1}{8} \times 360^\circ = 45^\circ
\]
A spiral with angle = 1/8 of revolution

\[
\frac{1}{8} \times 360^\circ = 45^\circ
\]
A spiral with angle = 1/8 of revolution

\[ \frac{1}{8} \times 360^\circ = 45^\circ \]
A spiral with angle = $1/8$ of revolution

$1/8 \times 360^\circ = 45^\circ$
A spiral with angle $= 1/8$ of revolution

$1/8 \times 360^\circ = 45^\circ$
A spiral with angle = 1/8 of revolution
1/8 x 360° = 45°

Leaves block each other.

Seeds crowd each other.

a lot of crowding

a lot of empty space
Instead of a 45° angle between seeds, plants use $\Phi \times 360^\circ = 582.5^\circ$ ....

$$\begin{align*}
582.5^\circ \\
- 360^\circ \\
\hline
= 222.5^\circ
\end{align*}$$

222.5° is called the Golden Angle.

It is 1.618 full turns.
How seeds are arranged in nature

Seed 1

Angle = 222.5°

0°

~222.5°

Angle = $\Phi$ revolutions
Seed 2

Angle = 222.5° × 2 revolutions

Angle = Φ × 2 revolutions
Seed 3

Angle = 222.5° × 3

Angle = \( \phi \times 3 \) revolutions
Seed 4  Angle = 222.5° x 4 revolutions
Seed 5
Angle = 222.5° x 5 revolutions
Seed 6  Angle = 222.5° × 6  revolutions
Seed n

Angle = \( 222.5^\circ \times n \)

(revolutions)

(distance from center to seed n = \( \sqrt{n} \))

Angle = \( \Phi \times n \)
Find spirals, find patterns...
Seed 8:
angle = $8 \times \Phi$
rotations

12.944 rotations
(same as 0.944 rotations)

$0.944 \times 360^\circ = 340^\circ$

...which is about 20° short of 360°
Every eighth seed: \( 8 \times \Phi \) rotations

\[
\sim 8 \times \frac{13}{8} \text{ rotations} = \sim 13 \text{ rotations}
\]

Nearly a whole number of rotations

Every thirteenth seed: \( 13 \times \Phi \) rotations

\[
\sim 13 \times \frac{21}{13} \text{ rotations} = \sim 21 \text{ rotations}
\]
Every 13th seed
13 x Φ rotations = 21.034...
rotations

= 12.4° past the previous seed in its spiral
Golden Spirals

This program is based on the arrangement of seeds in a sunflower. Points are plotted in an outward spiral, with a constant angle between consecutive points. The radius is proportional to the square root of the point number.

The sunflower uses the Golden Ratio (one of the preset angles) to achieve the most efficient arrangement of seeds. Other angles tend to produce noticeable "arms" where the points are clumped together.

When more than 400 points are plotted, the "show all spirals" and "show labels" options are disabled.
Why $\Phi$ and not some other number like $\pi$?

$\pi \approx \frac{22}{7} = 3.14159265358979...$

7 arms

Seed 7 is rotated $7 \times \pi$ revolutions = 21.9912 turns = about 357° = 3° from start...VERY CLOSE!

Compare: Seed 8 in the $\Phi$ spiral is 20° off...much worse for making spirals.
Seed 113 is rotated $113 \times \pi$ revolutions = 354.9999699 turns = about 359.989° = 0.011° from start!

$\pi \approx \frac{355}{113}$ (3.14159292...)

113 arms
Pi has GOOD approximations.
Good approximations make strong spiral arms.
Strong spiral arms cause crowding and empty space.

\[
\frac{22}{7}
\]
Good approximation!

\[
\frac{355}{113}
\]
GREAT approximation!

Good rational approximations = Bad seed distributions
The number with the worst possible approximations is...

The Golden Ratio is the **Most Irrational** Number...
The Golden Ratio gives the best seed placement!

Note: This can be analyzed with Hurwitz’s Theorem which gives a measure of how good or bad approximations are, and using **continued fractions** to generate convergents.
Every real number, rational or irrational, can be written as a continued fraction as follows:

\[ N = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e + \ldots}}}} \]

where \( a, b, c, d, e, \ldots \) are positive integers.
Here's the beginning of $\pi$ as a continued fraction.

$$\pi \approx 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{292 + \ldots}}}$$

The greater the integers, the better the rational approximations.
For fun...

\[
\pi = \left( \frac{339}{113} \right) + \frac{16}{1} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}
\]

\(\approx 3.14159292035...\)
These are the lowest we can go...

Φ has the worst approximations!
Mathematical Hypothesis:

Some plants twist uniformly as they grow.

The twist produces equal angles between leaves/branches/seeds.

The plant grows to a point which:
- maximizes light on leaves or
- minimizes stress (crowding) on seeds.
The resulting angle is *automatically* the Golden Ratio, and the Golden Ratio produces Fibonacci Numbers.
The mechanism may be

- active... feedback influences growth
- evolutionary... shaped over many generations
- or both.

The mechanisms differ in different plants, but the mathematical result is the same.
CRAZY IDEA:

Nature seeks the most chaos.
From this chaos, comes amazing order.
FIN